Mineral Physics I
Chapter 2. Elasticity
Section 3. Stress

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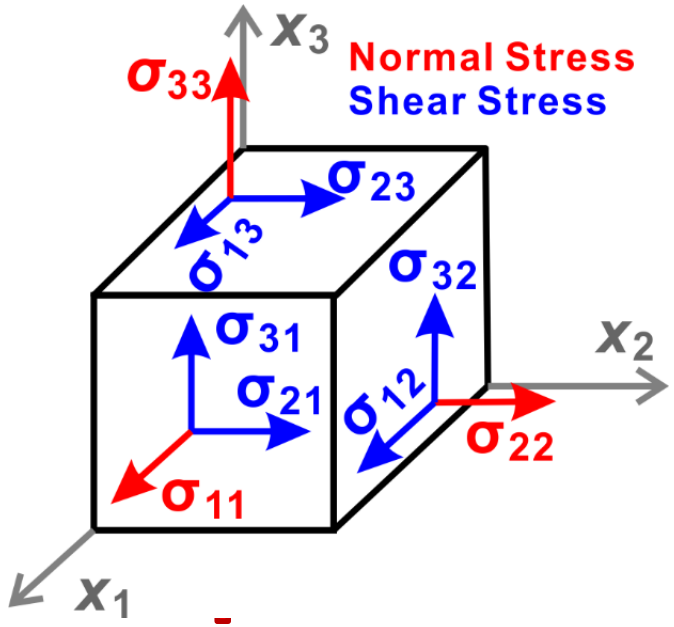
Stress tensor -1

- Stress: a force per unit area, applied across a small boundary.
  - The "boundary" can be defined on the surface and anywhere within the body.
  - The measure of a function to deform the body.
- Stress component in the x_i direction across the boundary normal to the x_j direction: σ_{ij}
  - Stress tensor (2nd-rank)
  - Normal stresses: $i = j$, normal to the boundary
  - Shear stresses: $i \neq j$, parallel to the boundary



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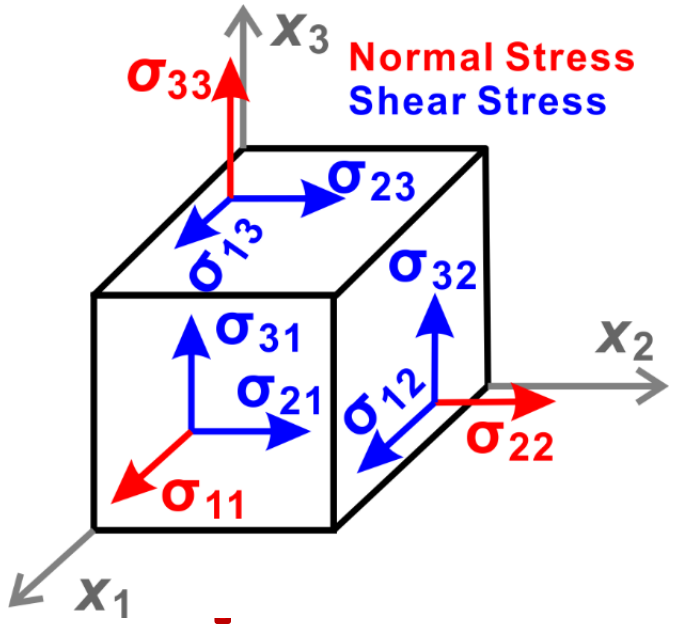
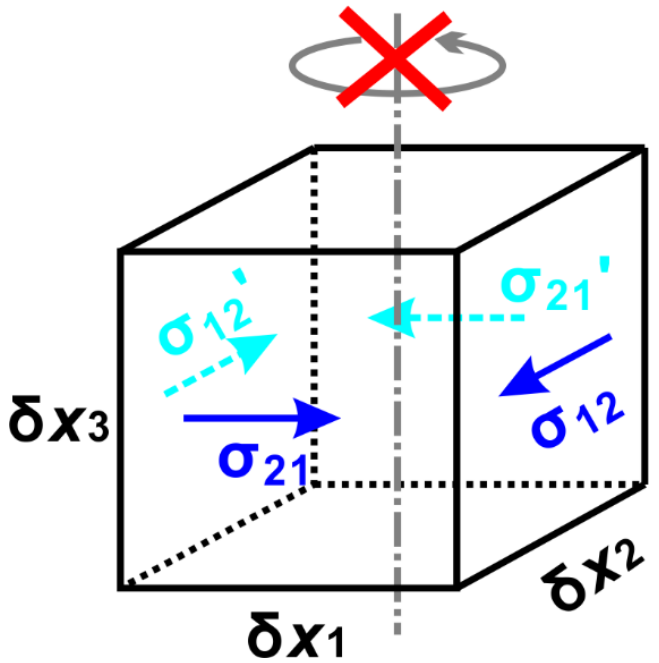


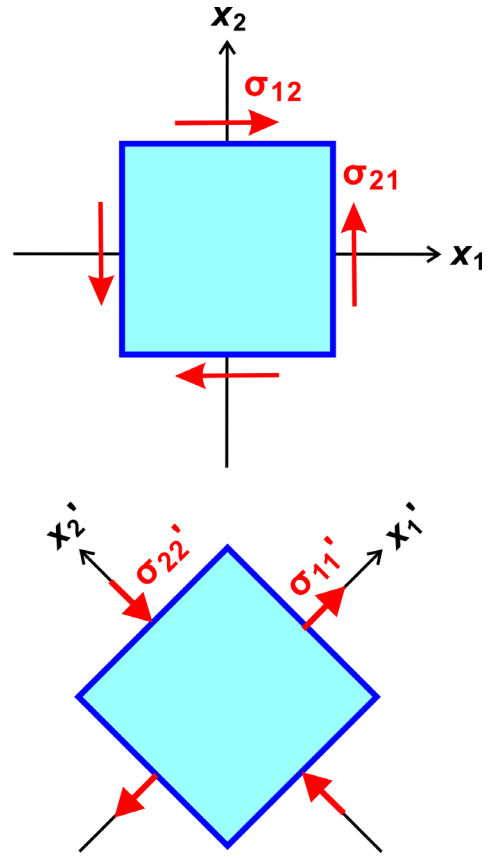
Stress tensor -2

- No rotation
  - $\sigma_{ij} = \sigma_{ji}$ (2.3.1)
    - ✓ Symmetric tensor
    - ✓ $[\sigma_{ij}] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$ (2.3.2)
- The signs of normal stresses
  - Outwards: positive
  - Inwards: negative



Principle stresses

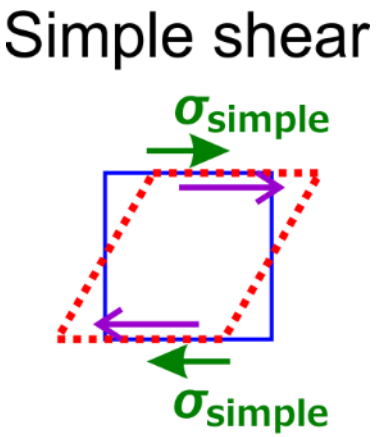
- As is for the strain tensor, the stress tensor can be orthogonalized, and the characteristic vectors are orthogonal due to symmetricity.
  - $A^{-1}[\sigma_{ij}]A = \begin{bmatrix} \sigma_{11}' & 0 & 0 \\ 0 & \sigma_{22}' & 0 \\ 0 & 0 & \sigma_{33}' \end{bmatrix}$ (2.2.3)
    - ✓ Principle stresses
  - Shear stresses are equivalent to combinations of normal stresses

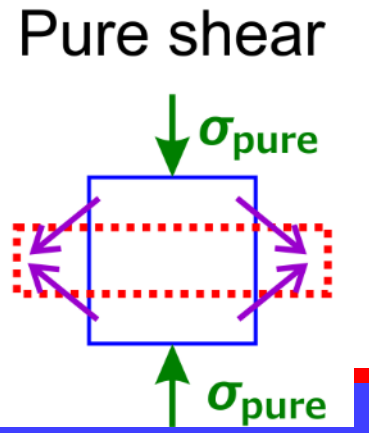


- Pressure, P:
  - Average of the (principle) normal stresses
  - $P = -(\sigma_{11}' + \sigma_{22}' + \sigma_{33}')/3$ (2.2.4)
    - ✓Positive stresses: outward
    - ✓Positive pressure: inward

Simple and pure shears

- **Simple shear**
  - Parallel planes in a body remain parallel and maintain a constant distance, while translating relative to each other
  - $[\sigma_{ij}] = \begin{bmatrix} 0 & \sigma_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (2.2.5)
    - ✓ Since the body rotates, the stress tensor is not symmetric
- **Pure shear**
  - Flattering of a body





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