Mineral Physics I
Chapter 4. Equation of State
Section 2. Birch-Murnaghan EOS

TOMOO KATSURA

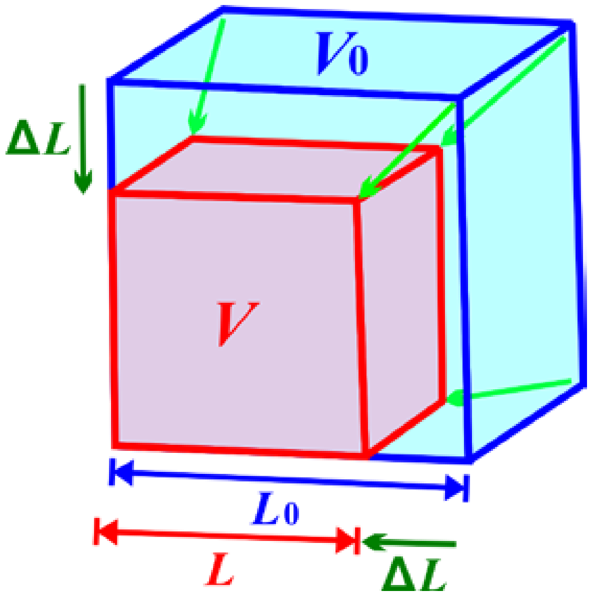
BAYERISCHES GEOINSTITUT, UNIVERSITY OF BAYREUTH, GERMANY

Finite strain theory

- Finite strain
  - It deals with large compression, which cannot be approximated by the infinite strain
    - The compression in Eq. (4.1.8) $P = K_{T0} \ln\left(\frac{V_0}{V}\right)$ is already large, though.
- Eulerian finite strain
  1. Compression is expressed by a state NOT before BUT after compression as a reference.
     - Compression is described not by V/V_0 but by V_0/V
       - When V goes to 0 $\Rightarrow$ P goes to ∞ $\leftarrow$ easily expressed using $\frac{V_0}{V} \rightarrow \infty$
  2. Discussion starts from change in squared length by compression.

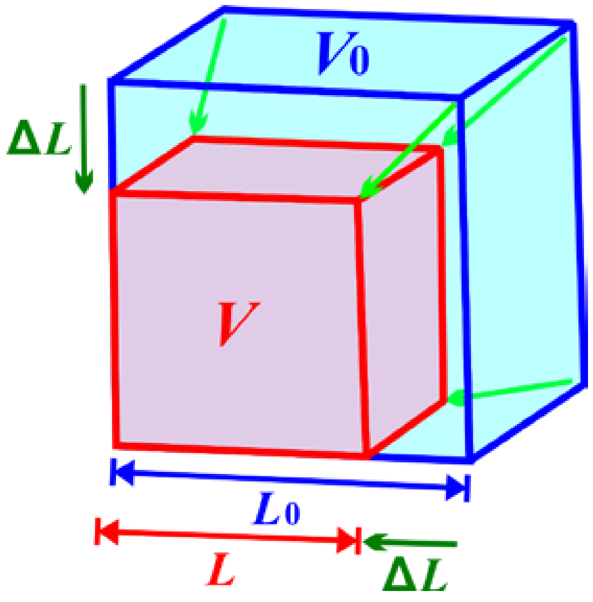
Eulerian finite strain -1

- A cube with edge lengths of L_0 in an unstrained state
  - Volume: $V_0 = L_0^3$ (4.2.1)
- Uniformly compressed to edge lengths of L
  - Volume; $V = L^3$ (4.2.2)
- Shortening (displacement) $\Delta L < 0$
  - $L = L_0 + \Delta L$ (4.2.3)
- Change in squared edge length
  - ${L_0}^2 - L^2 = (L - \Delta L)^2 - L^2 = \Delta L^2 - 2L\Delta L$ (4.2.4)



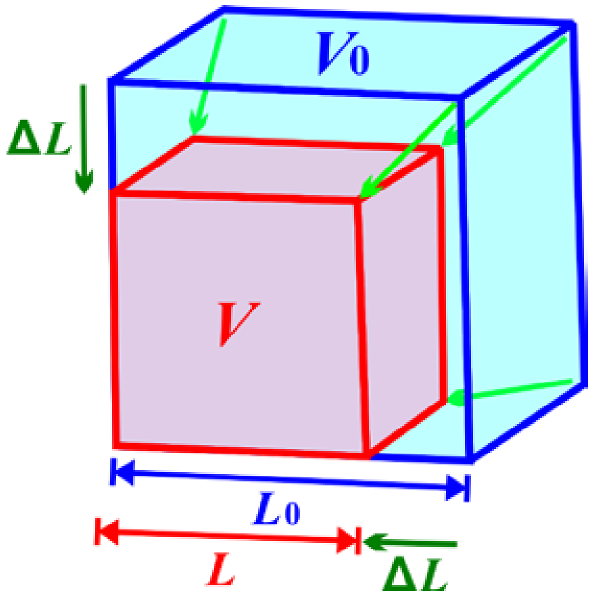
Eulerian finite strain -2

- The shortening should be proportional to the edge length
  - $\Delta L = cL$ (4.2.5)
- Eq. (4.2.4) is substituted to Eq. (4.2.5)
  - $L_0{}^2 - L^2 = \Delta L^2 - 2L\Delta L = (cL)^2 - 2LcL$
    $= (c^2 - 2c)L^2$ (4.2.6)
- Define the Eulerian finite strain as:
  - $f \equiv \frac{1}{2}c^2 - c$ (4.2.7)
- Using this definition, Eq. (4.2.6) becomes
  - $L_0{}^2 - L^2 = 2fL^2$



Eulerian finite strain -3

- ${L_0}^2 - L^2 = 2fL^2$
- ${L_0}^2 = (1 + 2f)L^2$
- ${L_0}^2/L^2 = (1 + 2f)$
- $L_0/L = (1 + 2f)^{1/2}$
- $V_0/V = {L_0}^3/L^3 = (1 + 2f)^{3/2}$
- $(V_0/V)^{2/3} = 1 + 2f$
- $f = \frac{1}{2}\left[(V_0/V)^{2/3} - 1\right]$ (4.2.8)
  - The Eulerian finite strain is expressed by the volume compression



2nd-order Birch-Murnaghan EOS -1

- P is V derivative of F at constant T
  - $P = -(\partial F/\partial V)_T$ (4.1.1)
- F is assumed to increase proportionally to the squared f with V decrease
  - $\Delta F \cong af^2$ (4.2.9)
    - a: constant
- V derivative of F is expressed by
  - $(\partial F/\partial V)_T = \{\partial(af^2)/\partial V\}_T = 2af(\partial f/\partial V)_T$
  - $P = -2af(\partial f/\partial V)_T$ (4.2.10)
- As is shown later:
  - $a = (9/2)K_{T_0}V_0$ (4.2.11)

2^{nd}-order Birch-Murnaghan EOS -2

q The V derivative of f is:

Ø $f = \frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right]$ (4.2.8)

Ø $\left(\frac{\partial f}{\partial V}\right)_T = \frac{1}{2}V_0^{\frac{2}{3}}\frac{\partial}{\partial V}\left(V^{-\frac{2}{3}}\right)_T = -\frac{1}{3}V_0^{\frac{2}{3}}V^{-\frac{5}{3}} = -\frac{1}{3V_0}\left(\frac{V_0}{V}\right)^{\frac{5}{3}}$ (4.2.12)

q The 2^{nd}-order Birch-Murnaghan EOS:

Ø $P = -2af\left(\frac{\partial f}{\partial V}\right)_T = -2\left(\frac{9}{2}K_{T0}V_0\right) \times \frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right] \times \left\{-\frac{1}{3V_0}\left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right\}$ (4.2.10)

Ø $P = \frac{3}{2}K_{T0}\left[\left(\frac{V_0}{V}\right)^{\frac{7}{3}} - \left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right]$ (4.2.13)

Derivation of $a = \frac{9}{2} K_{T_0} V_0$ -1

q V derivative of f at $P = 0$

Ø $\left(\frac{\partial f}{\partial V}\right)_T = -\left(\frac{1}{3V_0}\right)\left(\frac{V_0}{V}\right)^{\frac{5}{3}}$ (4.2.12)

Ø $\left(\frac{\partial f}{\partial V}\right)_{T,P=0} = -\frac{1}{3V_0}$ (4.2.14)

ü $V = V_0$ at $P = 0$

q V derivative of P at $P = 0$

Ø Definition of the isothermal bulk modulus: $K_T = -V\left(\frac{\partial P}{\partial V}\right)_T$

Ø $\left(\frac{\partial P}{\partial V}\right)_T = -\frac{K_T}{V}$

Ø $\left(\frac{\partial P}{\partial V}\right)_{T,P=0} = -\frac{K_{T0}}{V_0}$ (4.2.15)

Derivation of $a = \frac{9}{2}K_{T_0}V_0$ -2

q V derivative of P is expressed by V derivative of f at $P = 0$ from Eq. (4.2.10)

Ø $P = -2af\left(\frac{\partial f}{\partial V}\right)_T$ (4.2.10)

Ø $\left(\frac{\partial P}{\partial V}\right)_T = \left[\frac{\partial}{\partial V}\left\{-2af\left(\frac{\partial f}{\partial V}\right)_T\right\}\right]_T = -2a\left[\left(\frac{\partial f}{\partial V}\right)_T^2 + f\left(\frac{\partial^2 f}{\partial V^2}\right)_T\right]$ (4.2.15)

Ø $\left(\frac{\partial P}{\partial V}\right)_{T,P=0} = -2a\left(\frac{\partial f}{\partial V}\right)_{T,P=0}^2$ (4.2.16)

ü $f = 0$ at $P = 0$

q By substituting (4.2.14) and (4.2.15) into (4.2.16), we have;

Ø $-\frac{K_{T_0}}{V_0} = -2a\left(-\frac{1}{3V_0}\right)^2$

Ø $a = \frac{9}{2}K_{T_0}V_0$ (4.2.11)

3^{rd}-order Birch-Murnaghan EOS -1

- The concept of the 3rd-order Birch-Murnaghan EOS: almost identical to that of the 2nd-order
- Difference: change in F is expanded not to the 2nd-order but to the 3rd-order of f
  - $\Delta F \cong af^2 + bf^3$ (4.2.17)
- P: V derivative of F:
  - $P = -\left(\frac{\partial F}{\partial V}\right)_T$
  - $\left(\frac{\partial F}{\partial V}\right)_T = \left\{\frac{\partial}{\partial V}(af^2 + bf^3)\right\}_T = (2af + 3bf^2)\left(\frac{\partial f}{\partial V}\right)_T = 2af\left(1 + \frac{3b}{2a}f\right)\left(\frac{\partial f}{\partial V}\right)_T$
  - $P = -2af\left(1 + \frac{3b}{2a}f\right)\left(\frac{\partial f}{\partial V}\right)_T$ (4.2.18)

3rd-order Birch-Murnaghan EOS -2

- As shown in the next page
  - $\frac{3b}{2a} = \frac{3}{2}\left(K'_{T,0} - 4\right)$ (4.2.19)
- By substituting (4.2.8), (4.2.11), (4.2.12) and (4.2.19) into (4.2.18) $P = -2af\left(1 + \frac{3b}{2a}f\right)\left(\frac{\partial f}{\partial V}\right)_T$, we obtain the 3rd-order Birch-Murnaghan EOS:
  - $P = -2\left(\frac{9}{2}K_{T_0}V_0\right) \times \frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right] \times \left\{1 + \frac{3}{2}\left(K'_{T_0} - 4\right)\frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right]\right\} \times \left\{-\frac{1}{3V_0}\left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right\}$
  - $P = \frac{3}{2}K_{T_0}\left[\left(\frac{V_0}{V}\right)^{\frac{7}{3}} - \left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right] \times \left\{1 + \frac{3}{4}\left(K'_{T_0} - 4\right)\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right]\right\}$ (4.2.20)
    - The 3rd-order BM-EOS becomes identical to the 2nd-order BM-EOS when $K'_{T_0} = 4$

Derivation of $\frac{3b}{2a} = \frac{3}{2}\left(K'_{T_0} - 4\right)$ -1

- For simplicity, $m = \frac{3b}{2a}$
- Eq. (4.2.18) $P = -2af\left(1 + \frac{3b}{2a}f\right)\left(\frac{\partial f}{\partial V}\right)_T$ becomes

Ø $P = -2af(1 + mf)\left(\frac{\partial f}{\partial V}\right)_T$ (4.2.21)

- Differentiating (4.2.21) by V:

Ø $\left(\frac{\partial P}{\partial V}\right)_T = \left[\frac{\partial}{\partial V}\left\{-2af\left(\frac{\partial f}{\partial V}\right)_T(1 + mf)\right\}\right]_T$

$= -2a\left[\left(\frac{\partial f}{\partial V}\right)_T^2 + 2mf\left(\frac{\partial f}{\partial V}\right)_T^2 + f\left(\frac{\partial^2 f}{\partial V^2}\right)_T + mf^2\left(\frac{\partial^2 f}{\partial V^2}\right)_T\right]$ (4.2.22)

Derivation of $\frac{3b}{2a} = \frac{3}{2}\left(K'_{T_0} - 4\right)$ -2

- q Eq. (4.2.22) is once again differentiated by *V*:

Ø $\left(\frac{\partial^2 P}{\partial V^2}\right)_T = -2a\left[\frac{\partial}{\partial V}\left\{\left(\frac{\partial f}{\partial V}\right)_T^2 + 2mf\left(\frac{\partial f}{\partial V}\right)_T^2 + f\left(\frac{\partial^2 f}{\partial V^2}\right)_T + mf^2\left(\frac{\partial^2 f}{\partial V^2}\right)_T\right\}\right]_T$

$$\left(\frac{\partial^2 P}{\partial V^2}\right)_T = -2a\left[3\left(\frac{\partial f}{\partial V}\right)_T\left(\frac{\partial^2 f}{\partial V^2}\right)_T + 2m\left(\frac{\partial f}{\partial V}\right)_T^3 + 6mf\left(\frac{\partial f}{\partial V}\right)_T\left(\frac{\partial^2 f}{\partial V^2}\right)_T + f\left(\frac{\partial^3 f}{\partial V^3}\right)_T + \right.$$

Derivation of $\frac{3b}{2a} = \frac{3}{2}\left(K'_{T_0} - 4\right)$ -3

q 2nd V derivative of f

Ø $\left(\frac{\partial^2 f}{\partial V^2}\right)_T = \left(\frac{\partial}{\partial V}\left\{-\frac{1}{3}V_0^{\frac{2}{3}}V^{-\frac{5}{3}}\right\}\right)_T = \frac{5}{9V_0^2}\left(\frac{V_0}{V}\right)^{\frac{8}{3}}$ à $\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,P=0} = \frac{5}{9V_0^2}$ (4.2.25)

ü $V_0/V = 1$ at $P = 0$

q $K_T{}'$ and 2nd V derivative of P

Ø $K'_T = \left(\frac{\partial K_T}{\partial P}\right)_T = \left(\frac{\partial K_T}{\partial V}\right)_T\left(\frac{\partial V}{\partial P}\right)_T = \left(\frac{\partial\{-V(\partial P/\partial V)_T\}}{\partial V}\right)_T \frac{-V}{-V(\partial P/\partial V)_T} = \frac{V^2(\partial^2 P/\partial V^2)_T}{K_T} - 1$

Ø $\left(\frac{\partial^2 P}{\partial V^2}\right)_T = \frac{K_T}{V^2}(K'_T + 1)$

Ø $\left(\frac{\partial^2 P}{\partial V^2}\right)_{T,P=0} = \frac{K_{T_0}}{V_0^2}\left(K'_{T_0} + 1\right)$ (4.2.26)

Derivation of $\frac{3b}{2a} = \frac{3}{2}\left(K'_{T_0} - 4\right)$ -4

- By substituting (4.2.11) $a = \frac{9}{2}K_{T0}V_0$, (4.2.15) $\left(\frac{\partial f}{\partial V}\right)_{T,P=0} = -\frac{1}{3}V_0$, (4.2.23) $\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,P=0} = \frac{5}{9V_0^2}$, (4.2.24) $\left(\frac{\partial^2 P}{\partial V^2}\right)_{T,P=0} = \frac{K_{T_0}}{V_0^2}\left(K'_{T_0} + 1\right)$ into (4.2.26) $\left(\frac{\partial^2 P}{\partial V^2}\right)_{T,P=0} = -2a\left[3\left(\frac{\partial f}{\partial V}\right)_{T,P=0}\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,P=0} + 2m\left(\frac{\partial f}{\partial V}\right)^3_{T,P=0}\right]$
  - Ø $\frac{K_{T_0}}{V_0^2}\left(K'_{T_0} + 1\right) = -2\left(\frac{9}{2}K_{T0}V_0\right)\left[3\left(-\frac{1}{3V_0}\right)\frac{5}{9V_0^2} + 2m\left(-\frac{1}{3V_0}\right)^3\right]$
  - Ø $\frac{K_{T_0}}{V_0^2}\left(K'_{T_0} + 1\right) = \frac{2}{27V_0^3}\frac{9}{2}K_{T0}V_0(2m + 15)$ (4.2.27)
- By simplifying (4.2.27)
  - Ø $m = \frac{3b}{2a} = \frac{3}{2}\left(K'_{T_0} - 4\right)$ (4.2.19)

4^{th}-order Birch-Murnaghan EOS -1

- The concept of the 4^{th}-order Birch-Murnaghan EOS: almost identical to that of the 2^{nd}- and 3^{rd}-orders

- Change in F is expandeded to the 4^{th}-order of f
  - $\Delta F \cong a_2 f^2 + a_3 f^3 + a_4 f^4$ (4.2.28)

- P: V derivative of F:
  - $P = -\left(\frac{\partial F}{\partial V}\right)_T$
  - $\left(\frac{\partial F}{\partial V}\right)_T = \left\{\frac{\partial}{\partial V}(a_2 f^2 + a_3 f^3 + a_4 f^4)\right\}_T = (2a_2 f + 3a_3 f^2 + 4a_4 f^3)\left(\frac{\partial f}{\partial V}\right)_T$

$$= 2af\left(1 + \frac{3a_3}{2a_2} f + \frac{2a_4}{a_2} f^2\right)\left(\frac{\partial f}{\partial V}\right)_T \quad (4.2.29)$$

4^{th}-order Birch-Murnaghan EOS -2

- As shown in the next page
  - $\frac{2a_4}{a_2} = \frac{9K'^{2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ (4.2.30)
    - K''_{T_0}: the second pressure derivative of K_{T0}

- Similarly to the 3^{rd}-order Birch-Murnaghan EOS, we have 4^{th}-order EOS:
  - $P = -2\left(\frac{9}{2}K_{T_0}V_0\right) \times \frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right] \times \left\{-\frac{1}{3V_0}\left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right\}$
    $\times \left\{1 + \frac{3}{2}\left(K'_{T_0} - 4\right)\frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right] + \frac{9K'^{2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}\left\{\frac{1}{2}\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right]\right\}^2\right\}$
  - $P = \frac{3}{2}K_{T_0}\left[\left(\frac{V_0}{V}\right)^{\frac{7}{3}} - \left(\frac{V_0}{V}\right)^{\frac{5}{3}}\right] \times \left\{1 + \frac{3}{4}\left(K'_{T_0} - 4\right)\left[\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right] + \frac{9K'^{2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{24}\left\{\left(\frac{V_0}{V}\right)^{\frac{2}{3}} - 1\right\}^2\right\}$

(4.2.31)

Derivation of $\frac{2a_4}{a_2} = \frac{9K'^{2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ -1

q For simplicity, $m_2 = 2a_4/a_2$

q Eq. (4.2.18)

Ø $P = -(2a_2 f + 3a_3 f^2 + a_4 f^3)\left(\frac{\partial f}{\partial V}\right)_T$

$= -2a_2 f(1 + m_1 f + m_2 f^2)\left(\frac{\partial f}{\partial V}\right)_T$ (4.2.31)

q Differentiating (4.2.31) by V:

Ø $\left(\frac{\partial P}{\partial V}\right)_T = -2a_2\left\{(1 + 2m_1 f + 3m_2 f^2)\left(\frac{\partial f}{\partial V}\right)_T^2 + (f + m_1 f^2 + m_2 f^3)\left(\frac{\partial^2 f}{\partial V^2}\right)_T\right\}$

(4.2.32)

Derivation of $\frac{2a_4}{a_2} = \frac{9K'^{\,2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ -2

q Eq. (4.2.32) is once again differentiated by V:

$$Ø \left(\frac{\partial^2 P}{\partial V^2}\right)_T = -2a_2 \left\{ \begin{array}{c} (2m_1 + 6m_2 f)\left(\frac{\partial f}{\partial V}\right)_T^3 \\ +3\left(1 + 2m_1(2m_1 + 6m_2 f)\left(\frac{\partial f}{\partial V}\right)_T^3 + 3m_2 f^2\right)\left(\frac{\partial f}{\partial V}\right)_T\left(\frac{\partial^2 f}{\partial V^2}\right)_T \\ +(f + m_1 f^2 + m_2 f^3)\left(\frac{\partial^3 f}{\partial V^3}\right)_T \end{array} \right\} \qquad (4.2.33)$$

q Eq. (4.2.33) is once again differentiated by V:

$$Ø \left(\frac{\partial^3 P}{\partial V^3}\right)_T = -2a_2 \left\{ \begin{array}{c} 6m_2\left(\frac{\partial f}{\partial V}\right)_T^4 + 6(2m_1 + 6m_2 f)\left(\frac{\partial f}{\partial V}\right)_T^2\left(\frac{\partial^2 f}{\partial V^2}\right)_T \\ +3(1 + 2m_1 f + 3m_2 f^2)\left(\frac{\partial^2 f}{\partial V^2}\right)_T^2 + 4(1 + 2m_1 f + 3m_2 f^2)\left(\frac{\partial f}{\partial V}\right)_T\left(\frac{\partial^3 f}{\partial V^3}\right)_T \\ +(f + m_1 f^2 + m_2 f^3)\left(\frac{\partial^4 f}{\partial V^4}\right)_T \end{array} \right\} \qquad (4.2.34)$$

Derivation of $\frac{2a_4}{a_2} = \frac{9{K'_{T_0}}^2 - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ -3

- At $P = 0$, $V_0/V = 1$, $f = 0$:
- $\left(\frac{\partial^3 P}{\partial V^3}\right)_T =$

$$-2a_2\left\{\begin{array}{c} 6m_2\left(\frac{\partial f}{\partial V}\right)_T^4 + 6(2m_1 + 6m_2 f)\left(\frac{\partial f}{\partial V}\right)_T^2\left(\frac{\partial^2 f}{\partial V^2}\right)_T \\ +3(1 + 2m_1 f + 3m_2 f^2)\left(\frac{\partial^2 f}{\partial V^2}\right)_T^2 + 4(1 + 2m_1 f + 3m_2 f^2)\left(\frac{\partial f}{\partial V}\right)_T\left(\frac{\partial^3 f}{\partial V^3}\right)_T \\ +(f + m_1 f^2 + m_2 f^3)\left(\frac{\partial^4 f}{\partial V^4}\right)_T \end{array}\right\}$$

- $\left(\frac{\partial^3 P}{\partial V^3}\right)_{T,0} = -2a_2\left\{6m_2\left(\frac{\partial f}{\partial V}\right)_{T,0}^4 + 12m_1\left(\frac{\partial f}{\partial V}\right)_{T,0}^2\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,0} + 3\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,0}^2 + \right.$

(4.2.35)

Derivation of $\frac{2a_4}{a_2} = \frac{9{K'_{T_0}}^2 - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ - 4

q 3rd V derivative of f

Ø $f = \frac{1}{2}\left\{\left(\frac{V_0}{V}\right)^{2/3} - 1\right\}$

Ø $\left(\frac{\partial f}{\partial V}\right)_T = \partial/\partial V\left[(1/2)\left\{\left(\frac{V_0}{V}\right)^{2/3} - 1\right\}\right] = -\left(\frac{1}{3V_0}\right)\left(\frac{V_0}{V}\right)^{5/3}$

Ø $\left(\frac{\partial^2 f}{\partial V^2}\right)_T = \left(\partial/\partial V\left\{-\left(\frac{1}{3V_0}\right)\left(\frac{V_0}{V}\right)^{5/3}\right\}\right)_T = \left(\frac{5}{9V_0^2}\right)\left(\frac{V_0}{V}\right)^{8/3}$

Ø $\left(\frac{\partial^3 f}{\partial V^3}\right)_T = \left(\partial/\partial V\left\{\left(\frac{5}{9V_0^2}\right)\left(\frac{V_0}{V}\right)^{8/3}\right\}\right)_T = \left(-\frac{40}{27V_0^3}\right)\left(\frac{V_0}{V}\right)^{11/3}$ (4.2.36)

q at $P = 0$, $V_0/V = 1$

ü $\boxed{\left(\frac{\partial^3 f}{\partial V^3}\right)_{T,0}} = \boxed{-\frac{40}{27V_0^3}}$ (4.2.37)

Derivation of $\frac{2a_4}{a_2} = \frac{9K'^{\,2}_{T_0} - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ -5

- 3rd-V derivative of P expressed by K_T, K'_T and K_T''
  - $\left(\frac{\partial P}{\partial V}\right)_T = -\frac{K_T}{V}$, $\left(\frac{\partial^2 P}{\partial V^2}\right)_T = \frac{\partial}{\partial V}\left\{-\frac{K_T}{V}\right\} = \frac{K_T}{V^2}(K'_T + 1)$
  - $\left(\frac{\partial^3 P}{\partial V^3}\right)_T = \frac{\partial}{\partial V}\left\{\frac{K_T}{V^2}(K'_T + 1)\right\}_T = \left\{\frac{\partial P}{\partial V}\frac{\partial K_T}{\partial P}\right\}\frac{(K'_T+1)}{V^2} - 2\frac{K_T}{V^3}(K'_T + 1) + \frac{K_T}{V^2}\frac{\partial P}{\partial V}\frac{\partial K_T{}'}{\partial P}$
  - $\left(\frac{\partial^3 P}{\partial V^3}\right)_T = -\left(-V\frac{\partial P}{\partial V}\right)K'_T\frac{(K'_T+1)}{V^3} - 2\frac{K_T}{V^3}(K'_T + 1) - \frac{K_T}{V^3}\left(-V\frac{\partial P}{\partial V}\right)K''_T$
  - $\left(\frac{\partial^3 P}{\partial V^3}\right)_T = -\frac{K_T}{V^3}\left(K'^{\,2}_T + 3K'_T + 2 + K_T K''_T\right)$

- At zero pressure,
  - $\left(\frac{\partial^3 P}{\partial V^3}\right)_{T,0} = -\frac{K_{T,0}}{V_0^3}\left(K'^{\,2}_{T,0} + 3K'_{T,0} + K_{T,0}K''_{T,0} + 2\right)$ (4.2.38)

Derivation of $\frac{2a_4}{a_2} = \frac{9{K'_{T_0}}^2 - 63K'_{T_0} + 9K_{T_0}K''_{T_0} + 143}{6}$ - 6

- Eq. (4.2.35) becomes
  - $\left(\frac{\partial^3 P}{\partial V^3}\right)_{T,0} = -2a_2\left\{6m_2\left(\frac{\partial f}{\partial V}\right)_T^4 + 12m_1\left(\frac{\partial f}{\partial V}\right)_{T,0}^2\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,0} + 3\left(\frac{\partial^2 f}{\partial V^2}\right)_{T,0}^2 + \right.$

The 3rd-order EOS is not obtained by $K''_{T,0} = 0$.

Publications

English version

A Simple Derivation of the Birch–Murnaghan Equations of State (EOSs) and Comparison with EOSs Derived from Other Definitions of Finite Strain

by Tomoo Katsura [1,2,*] and Yoshinori Tange [3]

[1] Bayerisches Geoinstitut, University of Bayreuth, 95440 Bayreuth, Germany
[2] Center of High Pressure Science and Technology Advanced Research (HPSTAR), Beijing 100094, China
[3] Japan Synchrotron Radiation Research Institute (JASRI), Kouto 679-5198, Japan
[*] Author to whom correspondence should be addressed.

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Japanese version

解説

バーチ・マーナハン状態方程式の平易な導出と，他の状態方程式との比較

A Plain Derivation of Birch-Murnaghan Equations of State, and Comparison with Other Equations of State

桂　智男[1,2,*]　　丹下　慶範[3]

Tomoo KATSURA[1,2,*]　　Yoshinori TANGE[3]

[1] バイロイト大学バイエルン地球科学研究所
Bayerisches Geoinstitut, University of Bayreuth, 95440 Bayreuth, Germany

[2] 北京高圧科学研究中心
Center for High Pressure Science & Technology Advanced Research, Bldg. # 8E, ZPark, 10 Xibeiwang East Rd, Haidian District, Beijing, 100094, P.R. China

[3] 〒679–5198　兵庫県佐用郡佐用町光都 1 丁目 1–1　高輝度光科学研究センター
Japan Synchrotron Radiation Research Institute (JASRI), 1–1–1 Kouto, Sayo-cho, Sayo-gun, Hyogo 679–5198

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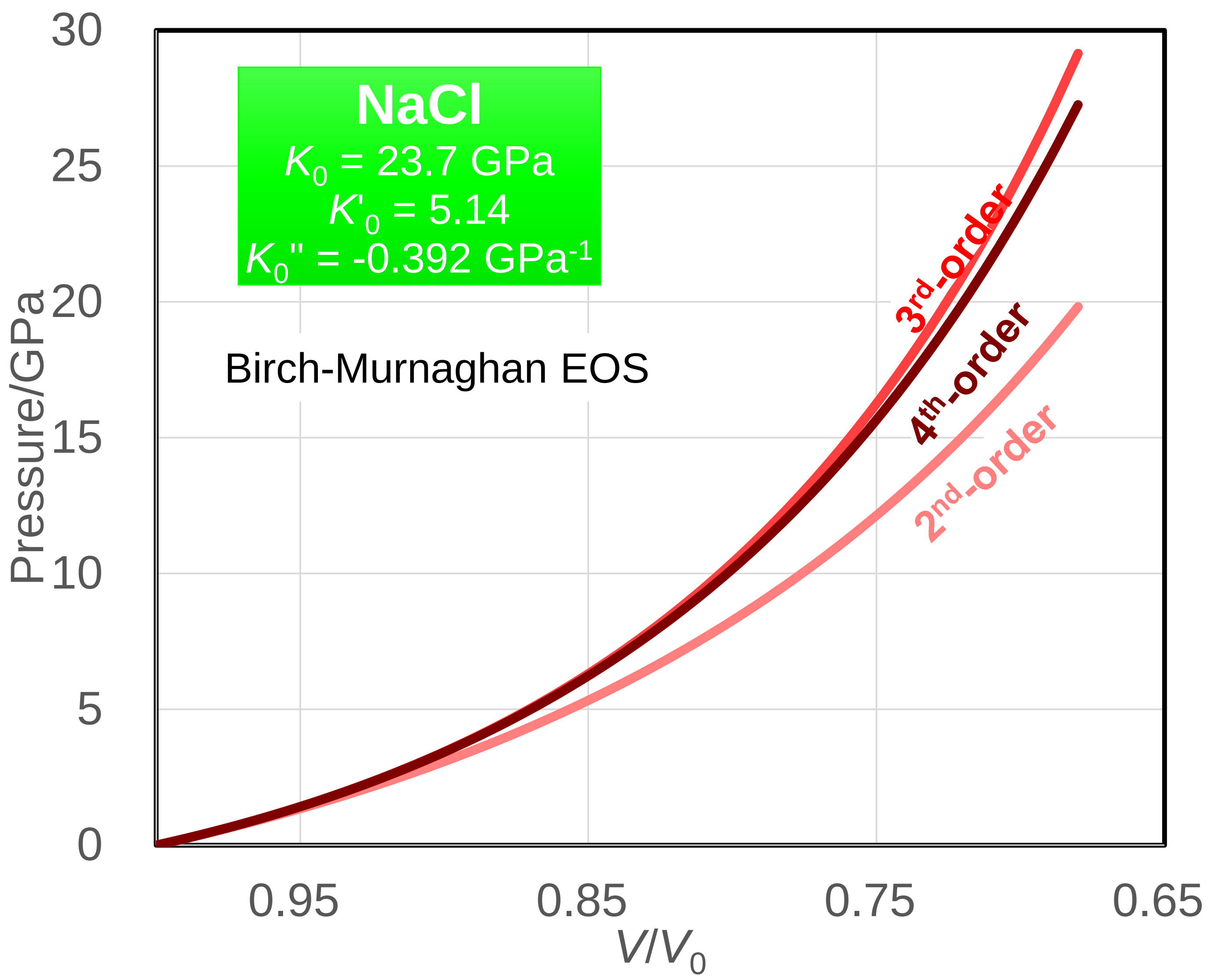
Compression of NaCl based on BM-EOS

Very low K_0 = 23.7 GPa

K'_0 larger than 4 (5.14)

negative K_0''

Relatively low pressure values even at high compression



Compression of MgO based on BM-EOS

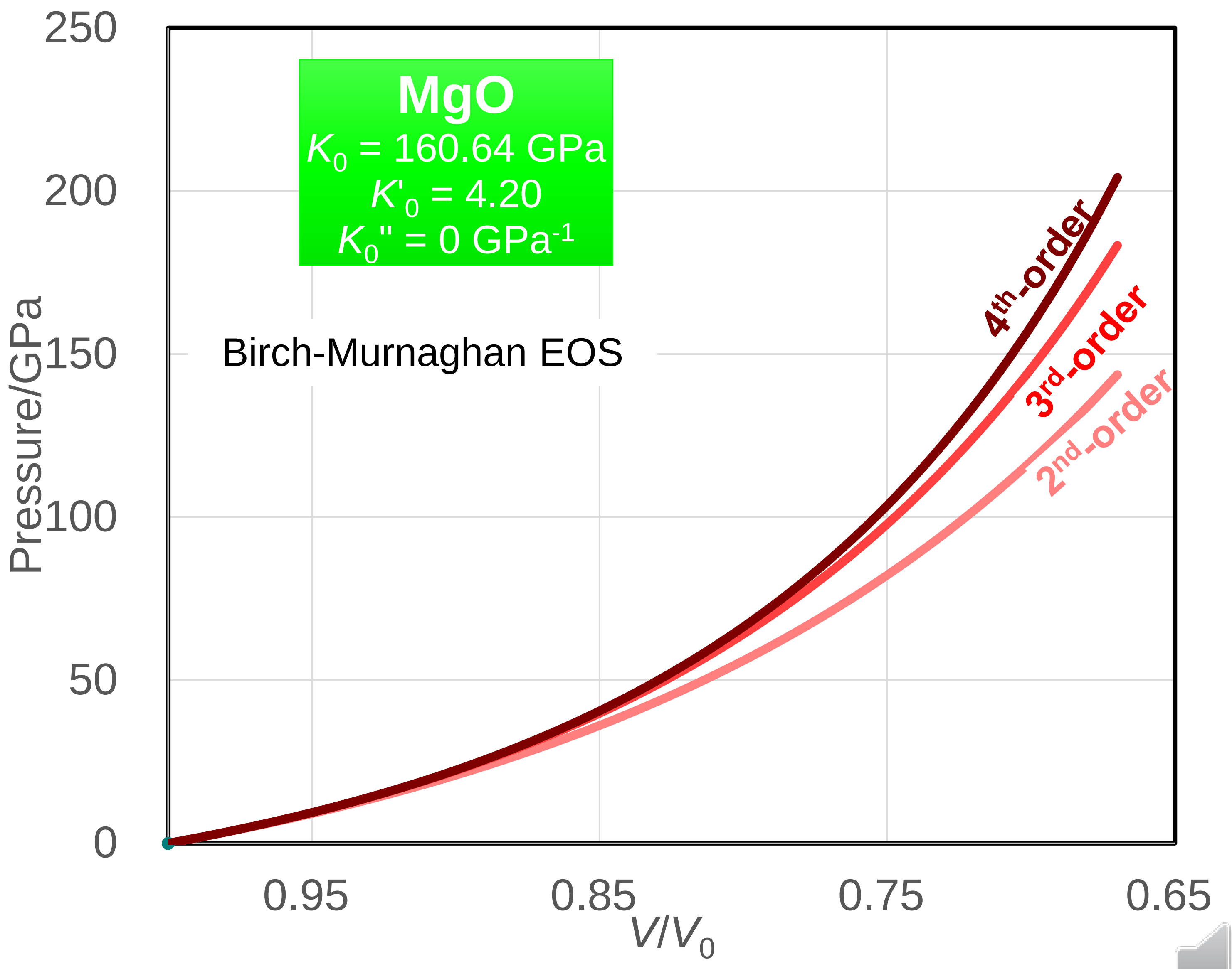
Relatively high K_0 = 160 GPa

K'_0 slightly larger than 4 (4.20)

No K_0" data

- K_0"=0 does not mean that 4^{th} EOS becomes 3^{rd} EOS

Relatively high pressure values by large compression



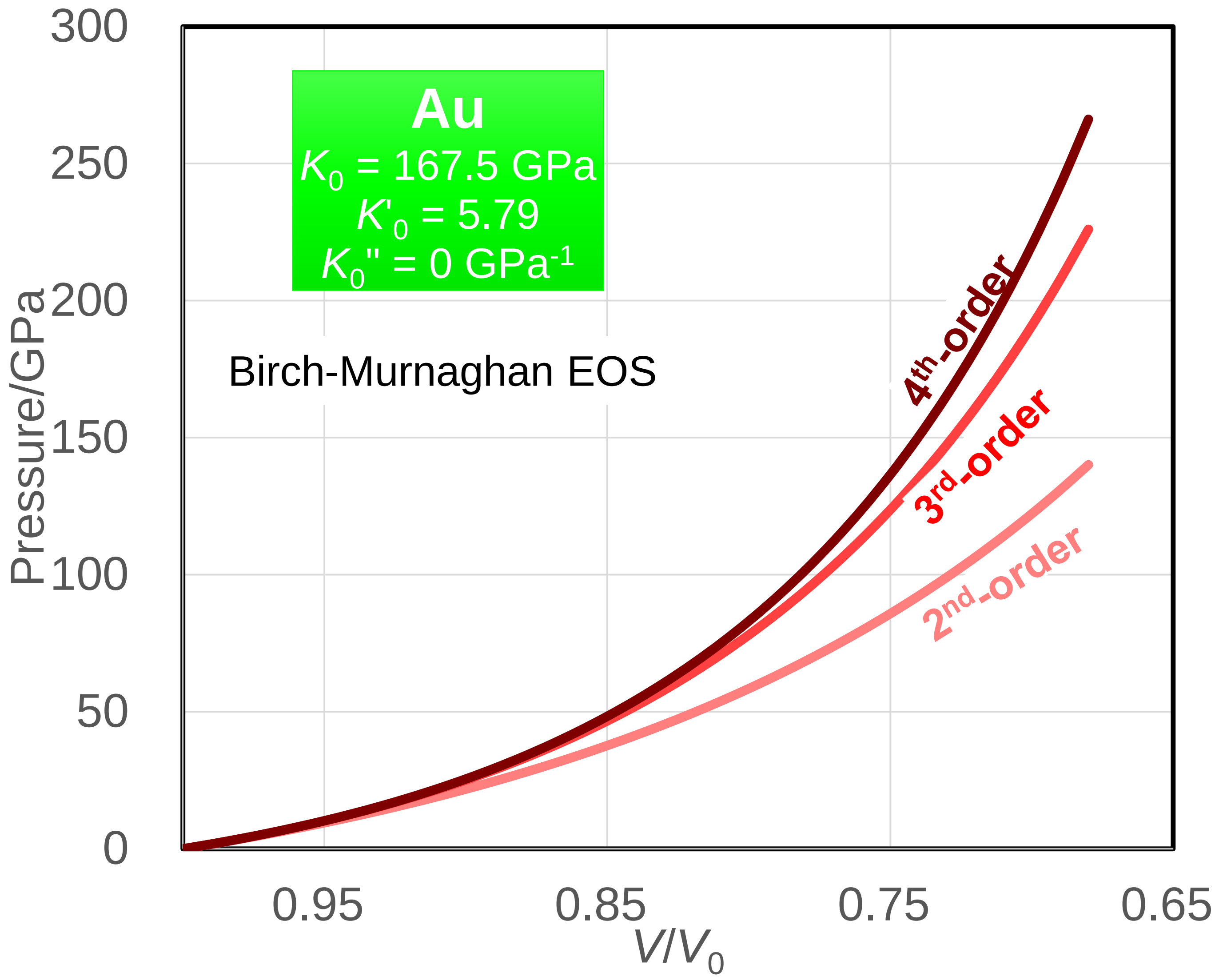
Compression of Au based on BM-EOS

Relatively high K_0 = 168 GPa

K'_0 larger than 4 (5.8)

No K_0" data

Relatively high pressure values by large compression



Mineral Physics I
Chapter 4. Equation of state
Section 2. Birch-Murnaghan equation of state

End