5. Heat transfer properties

3. Radiative thermal conductivity

3.1 Thermal radiation

All matter with a temperature (T) greater than 0 K emits thermal radiation which is electromagnetic radiation generated by the thermal motion of particles in matter. The internal energy of lattice vibration is converted to electromagnetic waves to be released to space as thermal radiation.

The basic features of thermal radiation are becoming stronger with increasing temperature (T) as shown in Figure 3.1, and continuous spectra. Color, or wavelength distribution of energy density, changes from red to white increasing temperature (T). For instance, a matter with 5,500 K emits large amounts of thermal radiation and its peak position is located at ~500 nm, while a matter with 3,500 K emits smaller amounts of thermal radiation than that with 5,500 K and its peak position is located at ~800 nm, resulting in red appearance to the human eyes.

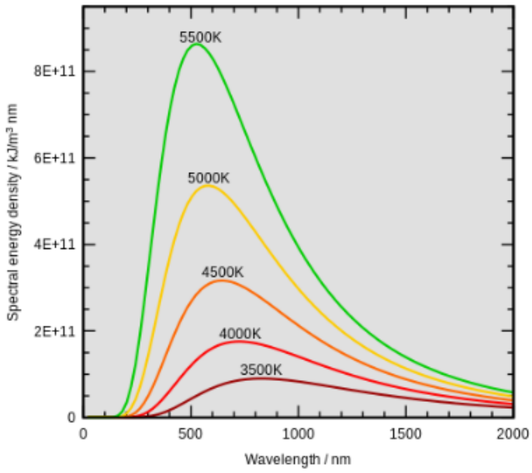

Fig. 3.1 Thermal radiation of materials with different temperatures. The peak wavelength and total amounts of thermal radiation vary with the temperature of a matter. (Cited from thermal radiation in Wikipedia)

3.2 Specific intensity

To describe characteristics of thermal radiation, some a variety of radiometric quantities are set as SI radiometry units with different unit or dimension such as radiance and radiant emittance. Radiance is the radiant energy flux from a surface per unit solid angle per unit area of the radiating surface. Radiant emittance is the radiant energy flux emitted by a surface per unit area and it is an old term that has now been replaced by radiant exitance. To mention frequency or wavelength distribution of radiometric properties, spectral radiance and spectral energy density are used. Spectral radiance is radiance at a given frequency or wavelength. Spectral energy density is stored energy per unit volume at a given frequency or wavelength.

Spectral radiance is also described as specific (radiative) intensity $I_\nu(cos\theta)$ which can be expressed in base SI units as $W \cdot m^{-2} \cdot sr^{-1} \cdot Hz^{-1}$. This nomenclature, specific intensity, is dominantly used in astrophysics and astronomy. Figure 3.2 shows the geometry for the definition of specific intensity. The amount of energy (dE_ν) transported through a surface area (dA) is proportional to the length time (dt), frequency width ($d\nu$), solid angle ($d\omega$), and the projected unit surface area ($cos\theta dA$).

$$dE_\nu = I_\nu(cos\theta)cos\theta dA d\omega d\nu dt \qquad (5.3.1)$$

where $I_\nu(\cos\theta)$ is the specific intensity (J $m^{-2}sr^{-1}Hz^{-1}s^{-1}$), dA is a surface area, $cos\theta dA$ is the projected unit surface area, dt is the length time, $d\nu$ is a frequency width, $d\omega$ is a solid angle, and dE_ν is the amount of energy transported through a surface area (dA).

$$I_\lambda = \frac{c}{\lambda^2} I_\nu$$
$$d\nu = -\frac{c}{\lambda^2} d\lambda \qquad \because I_\lambda d\lambda = I_\nu d\nu, \quad \nu = \frac{c}{\lambda} \tag{5.3.2}$$

Where ν is frequency, λ is wavelength, and c is speed of light.

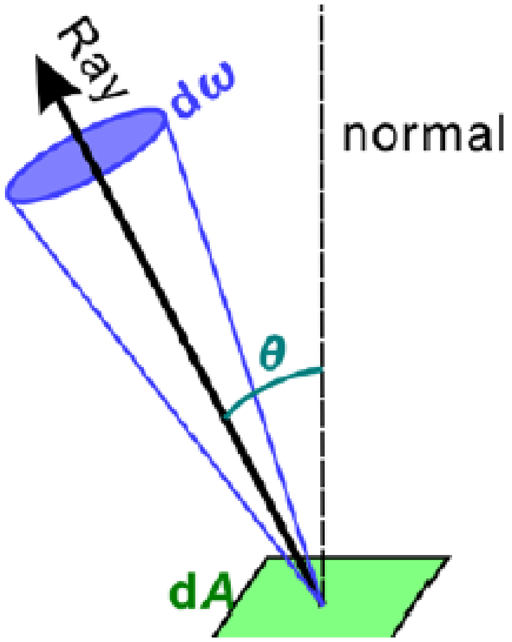

Fig. 3.2 The geometry for the definition of specific (radiative) intensity.

3.3 Radiative flux

Radiative flux is the amount of energy flows through a surface element dA. This quantity "radiative flux" is often used in astronomy and meteorology to discuss the amount of power radiated through a given area such as planetary surfaces. Photons or other elementary particles The SI unit is W/m^2.

$$dE_\nu \propto I_\nu cos\theta d\omega \tag{5.3.0}$$

Where E_ν is the amount of energy, $I_\nu cos\theta$ is the specific intensity, and $d\omega$ is the solid angle.

Flux, F_ν, is described as integration of the specific intensity over the half solid angle. If we consider a hemisphere as shown in Figure 3.3.

$$F_\nu = \int_{2\pi} I_\nu cos\theta d\omega$$
$$= \int_0^{2\pi} \int_0^{\pi/2} I_\nu cos\theta sin\theta d\theta d\Phi \tag{5.3.3}$$

where $I_\nu(\cos\theta)$ is the specific intensity (J $m^{-2}sr^{-1}Hz^{-1}s^{-1}$), $d\omega$ is a solid angle, and dE_ν is the amount of energy transported through a surface area (dA).

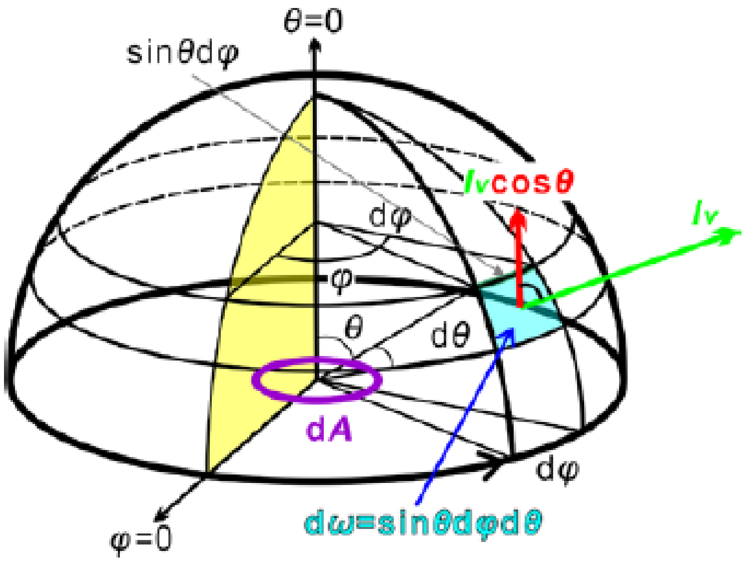

Fig. 3.3 A schematic image of radiative flux

3.4 Planck's law of thermal radiation

Planck's law is an essential equation in electromagnetic physics to describe the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T (Fig. 3.4).

$$B_\nu(\nu, T)d\nu = \frac{2h\nu^3}{c^2}\frac{1}{exp(h\nu/k_B T) - 1}d\nu \qquad (5.3.4)$$

where h is the Planck's constant, c is the speed of light, and k_B is the Boltzmann's constant.

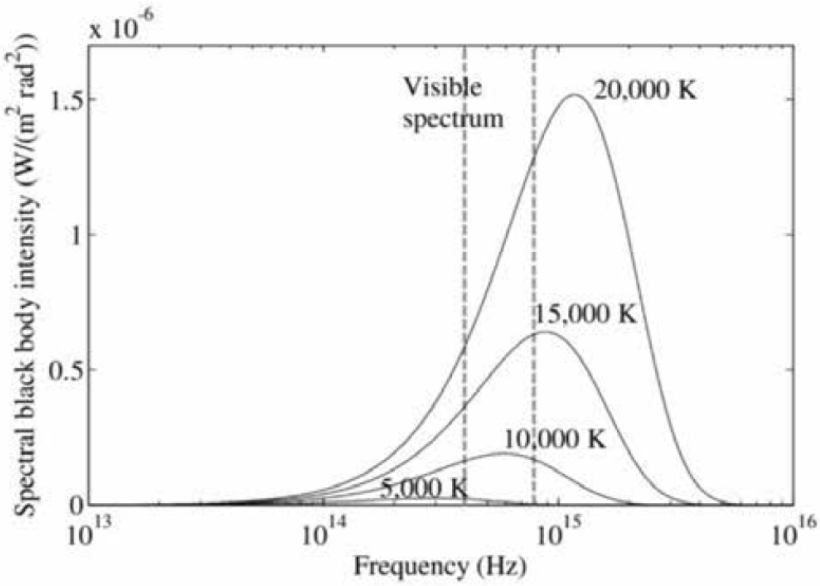

Fig. 3.4 Spectral black body intensity of matters with different temperatures. Planck's law describes black-body radiation. Spectral intensity becomes higher with increasing temperature and the peak frequency shifts higher frequency (shorter wavelength) with increasing temperature.

There are many ways to describe Planck's law such as a description of spectral radiance form ($Jm^{-2}sr^{-1}s^{-1}$) with frequency:

$$B_v(\nu, T)dv = \frac{2h\nu^3}{c^2}\frac{1}{exp(h\nu/k_BT) - 1}dv \qquad (5.3.4)$$

for description of spectral radiance form ($Jm^{-2}sr^{-1}s^{-1}$) with wavelength:

$$B_v(\lambda, T)dv = \frac{2hc^2}{\lambda^5}\frac{1}{exp(hc/\lambda k_BT) - 1}dv \qquad (5.3.5)$$

considering the equation below (the details will be given in the later subsection):

$$u(T) = \frac{4\pi}{c}B(T) \qquad (5.3.6)$$

for description of spectral energy density form (Jm^{-3}) with frequency:

$$u_v(\nu, T)dv = \frac{8\pi h\nu^3}{c^3}\frac{1}{exp(h\nu/k_BT) - 1}dv \qquad (5.3.7)$$

For description of spectral energy density form (Jm^{-3}) with wavelength:

$$B_v(\lambda, T)dv = \frac{8\pi hc}{\lambda^5}\frac{1}{exp(hc/\lambda k_BT) - 1}dv \qquad (5.3.8)$$

where ν is frequency, λ is wavelength, h is the Planck's constant, c is the speed of light, and k_B is the Boltzmann's constant.

3.5 Stefan-Boltzmann law

Stefan-Boltzmann law describes the total E radiated per unit surface area of a black-body across all frequency per unit *t*, which is directly proportional to the fourth power of the thermodynamic temperature T^4. For radiance:

$$L = \frac{\sigma}{\pi}T^4 \qquad (5.3.18)$$

Where T is temperature. For radiant emittance:

$$j* = \sigma T^4 \qquad (5.3.19)$$

For energy density:

$$U = \frac{4\sigma}{c}T^4 \qquad (5.3.20)$$

Here the constant of proportionality in the Stefan-Boltzmann law σ is called as Stefan-Boltzmann constant. The constant is defined in terms of other fundamental constants in SI units as:

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67 \ldots \times 10^{-8}\, Wm^{-2}K^{-4} \qquad (5.3.21)$$

Where k_B is the Boltzmann constant, h is the Planck constant, and c is the speed of light.

Radiation laws in a transparent medium

The velocity of light in a medium is slower than in vacuum. By the speed of light in vacuum and the phase velocity of light in the medium, the refractive index is defined as:

$$n = \frac{c}{q} \qquad (5.3.24)$$

Where n is the refractive index, c is the speed of light in vacuum, and q is the speed of light in the medium. Considering the speed of light in the medium, the Planck's law (5.3.4) can be replaced by:

$$B_\nu^{med}(\nu,T) = \frac{2h}{q^2}\frac{\nu^3}{exp(h\nu/k_BT)-1} = \left(\frac{c}{q}\right)^2\frac{2h}{c^2}\frac{\nu^3}{exp(h\nu/k_BT)-1_\nu} = n^2 B_\nu(\nu,T) \qquad (5.3.25)$$

where ν is wavenumber, T is the temperature, q is the speed of light in a medium, h is the Planck's constant, k_B is the Boltzmann's constant, and c is the speed of light. Stefan-Boltzmann law (5.3.18) can also be replaced by:

$$L = \int_0^\infty n^2 B_\nu^{med}(\nu,T)d\nu = \langle n^2\rangle \frac{\sigma}{\pi}T^4 \qquad (5.3.26)$$

where ν is wavenumber, T is the temperature, n is the refractive index, and σ is the Stefan-Boltzmann constant.

$$\langle n^2\rangle = \frac{\int_0^\infty n^2 B_\nu^{med}(\nu,T)d\nu}{\int_0^\infty B_\nu^{med}(\nu,T)d\nu} \qquad (5.3.27)$$

suggesting the radiated energy in a medium is higher than vacuum, proportionally to $\langle n^2\rangle$.

Thermal resistance of radiative heat transfer

Materials resist a heat flow and cause a temperature difference. This heat property is defined as thermal resistance, which is the reciprocal of thermal conductivity. Suppose perfectly transparent material. Thermal radiation should go through the body without any interaction, therefore, the body is not heated and no thermal conductivity is caused by thermal radiation. Actual materials absorb light to some extent and energy is transferred from one part of the body to another part of the body. The part at higher temperature than other parts emits more light as thermal radiation. The temperature of this hotter part decreases by losing energy, whereas that of the neighboring part increases by absorbing the emitted light. How much matter takes absorbs the energy is referred to as (optical) absorption, which is an essential property for the radiative heat transfer.

Absorbance, absorption coefficient, and absorptivity

The intensity of light passing through a semi-transparent body I_{tr} decreases exponentially in a body and it is defined as:

$$I_{tr}(s) = I_0 exp(-\alpha s) \approx I_0(1-\alpha s) \qquad (5.3.28)$$

where I_{tr} is the intensity of transmitted light, I_0 is the intensity of incident light, α is the absorption coefficient [m^{-1}] and s is the path. How much changes in the intensity is described as:

$$dI = Id(exp(-\alpha s)) \approx Id(1-\alpha s) = -\alpha I ds \qquad (5.3.29)$$

where α is the absorption coefficient [m^{-1}] and s is the path.

Absorbance A is referred to as the logarithm of the ratio of the incident to transmitted intensity.

$$A = ln(\frac{I_0}{I_{tr}}) = ln(\frac{I_0}{I_0 exp(-\alpha s)}) = \alpha s \qquad (5.3.30)$$

where A is the absorbance. In the equation (5.3.29) the natural logarithm (ln) is used, while actual measurements using FTIR often prefer to use common logarithm (log10).

The intensity of the light absorbed by the body I_{abs} is derived as:

$$I_{abs} = I_0 - I_{tr} = I_0 - I_0 exp(-\alpha s) \approx I_0(1 - exp(-\alpha s)) \qquad (5.3.31)$$

Absorptivity ξ, the fraction of total incident radiation which is absorbed by a body, is defined as:

$$\xi = \frac{I_{abs}}{I_0} = \frac{I_0 - I_{tr}}{I_0} \tag{5.3.32}$$

$$\xi = \frac{I_0 - I_0 exp(-\alpha s)}{I_0} = 1 - exp(-\alpha s) \tag{5.3.33}$$

where ξ is the absorptivity, α is the absorption coefficient [m^{-1}] and s is the path.

Photon mean free path

Photon mean free path Λ_{pht} is the average length of transmission of the light. A photons encounters another photon only once while traveling the length of photon mean free path and the photon substantially changes its direction or energy by the collision with other photons. The number of photon is proportional to the light intensity. the intensity of light transmitted over the pass is described in the equation (5.3.28).

$$\begin{aligned}\Lambda_{pht} &= \frac{\int_0^{\infty} sI_0 exp(-\alpha s)\, ds}{\int_0^{\infty} I_0 exp(-\alpha s)\, ds} \\ &= \frac{\frac{1}{\alpha^2}\int_0^{\infty}(-\alpha)xI_0 exp(-\alpha s)\, ds}{-\frac{1}{\alpha}\int_0^{\infty} exp(-\alpha x)(-\alpha)\, ds} \\ &= -\frac{1}{\alpha}\frac{\int_0^{\infty} ze^{-z}dz}{\int_0^{\infty} e^{-z}dz} \\ &= \frac{1}{\alpha}\end{aligned} \tag{5.3.34}$$

Where Λ_{pht} is the photon mean free path, I_0 is the intensity of incident light, α is the absorption coefficient [m^{-1}] and s is the path. The equation (5.3.34) indicates that the photon mean free path is the reciprocal absorption coefficient, therefore, the equation (5.3.28) can be replaced with photon mean free path as:

$$I_{tr}(s) = I_0 exp(-s/\Lambda_{pht}) \tag{5.3.35}$$

where I_{tr} is the intensity of transmitted light, I_0 is the intensity of incident light, s is the path, Λ_{pht} is the photon mean free path.

The equation of radiative heat transfer

To calculate radiative heat transfer from matter, suppose the body shown in Figure 3.5.

$$dE_\nu^{em} = \epsilon_\nu dV d\omega d\nu dt = \epsilon_\nu dA ds d\omega d\nu dt \tag{5.3.36}$$

where ϵ_ν is the emission coefficient [$Jm^{-3}sr^{-1}Hz^{-1}s^{-1}$], and other properties shown in Figure 3.5. With emission coefficient ϵ_ν, emissivity ε_ν [m^{-1}] is described as:

$$\epsilon_\nu = \varepsilon_\nu B_\nu \tag{5.3.36}$$

From the equation (5.3.23), the equation below is supported.

$$dI = -\alpha I ds \tag{5.3.23b}$$

Thus, the energy removed by absorption is described as:

$$dE_\nu^{abs} = dI_\nu^{abs} dA d\omega d\nu dt = -\alpha_\nu dI_\nu dA ds d\omega d\nu dt \tag{5.3.37}$$

Here the net change in specific intensity is described as:

$$dI_\nu = -\alpha_\nu dI_\nu ds + \epsilon_\nu ds \tag{5.3.38}$$

thus, the equation of radiative heat transfer is rephrased as:

$$\frac{dI_\nu}{ds} = -\alpha_\nu dI_\nu + \epsilon_\nu \tag{5.3.39}$$

and this differential equation describes the flow of radiation through matter.

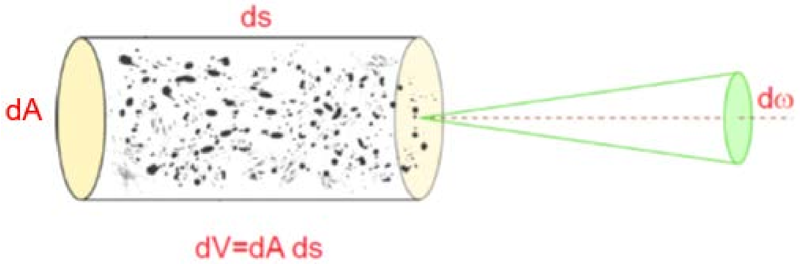

Fig. 3.5 a schematic image of the body with A of the cross-sectional area and s of the length.

Plane-parallel symmetry

Consider the equation of radiative heat transfer which go through in one direction as shown in Figure 3.6 with the concept of plane-parallel symmetry. In Figure 3.6, temperature gradient is vertical to the direction of x. The angle θ is defined as the angle between the direction of dx and ds.

$$dx = cos\theta ds = \mu ds \tag{5.3.40}$$

$$\mu = cos\theta \tag{5.3.41}$$

$$\frac{d}{ds} = \mu\frac{d}{dx} \tag{5.3.42}$$

With (5.3.39) and (5.3.42), the equation of radiative heat transfer is expressed as:

$$\frac{dI_\nu(\mu,x)}{dx} = -\alpha_\nu(x)I_\nu(\mu,x) + \epsilon_\nu(x) \tag{5.3.43}$$

where I_ν is the specific intensity, α_ν is the absorption coefficient, and ϵ_ν is the emission coefficient.

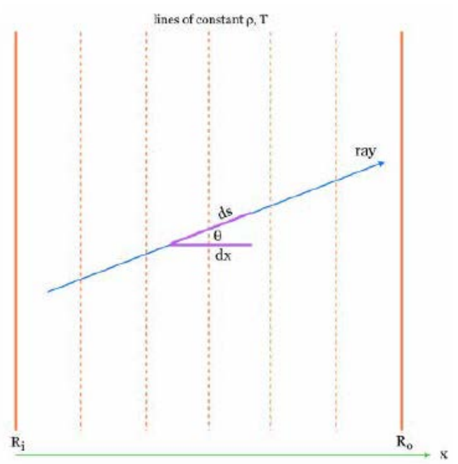

Fig.3.6 Radiative heat transfer which propagates in one direction.

Source function & local thermal equilibrium

In plane-parallel symmetry, the equation (5.3.43) can be rephrased as:

$$\frac{\mu}{\alpha_\nu(x)}\frac{dI_\nu(\mu,x)}{dx} = -I_\nu(\mu,x) + \frac{\epsilon_\nu(x)}{\alpha_\nu(x)} = S_\nu(x) - I_\nu(\mu,x) \qquad (5.3.44)$$

where I_ν is the specific intensity, α is the absorption coefficient, ϵ_ν is the emission coefficient, and S_ν is the source function defined as:

$$S_\nu(x) = \frac{\epsilon_\nu(x)}{\alpha_\nu(x)} \qquad (5.3.45)$$

where ϵ_ν is the emission coefficient. Considering local thermal equilibrium (LTE), Kirchhoff's law, which describes balance of emission and absorption, can help the understanding of radiative heat transfer as:

$$\varepsilon_\nu(x) = \frac{\epsilon_\nu(x)}{B_\nu(x)} \qquad (5.3.46)$$

thus,

$$\xi_\nu(x) \approx \alpha_\nu(x) \qquad (5.3.47)$$

thus, source function can be described as:

$$S_\nu(x) = \frac{\epsilon_\nu(x)}{\alpha_\nu(x)} = \frac{\varepsilon_\nu(x)B_\nu(x)}{\alpha_\nu(x)} \approx B_\nu(x) \qquad (5.3.48)$$

where S_ν is the source function, ϵ_ν is the emission coefficient, α is the absorption coefficient, and B_ν is the black-body function.

Radiative thermal conductivity

Consider radiative thermal conductivity under dT/dx.

$$I_\nu(\mu,x) = B_\nu^{med}(x) - \frac{\mu}{\alpha_\nu}\frac{dI_\nu(\mu,x)}{dx} \approx B_\nu^{med}(x) - \frac{\mu}{\alpha_\nu}\frac{\partial B_\nu^{med}(\nu,T)}{\partial x} \qquad (5.3.49)$$

thus,

$$F_\nu = \int_{4\pi} I_\nu \mu d\Omega = \int_{4\pi} B_\nu^{med}(\nu,T) d\Omega - \int_0^{2\pi} d\Phi \int_0^{\pi} \frac{\mu}{\alpha_\nu}\frac{\partial B_\nu^{med}(\nu,T)}{\partial x} cos\theta sin\theta d\theta \qquad (5.3.49b)$$

In the expansion above, $\int_{4\pi} \mathrm{B}_\nu^{med}(\nu,\mathrm{T})d\Omega \approx 0$, $\mu = cos\theta$ and $d\mu = -sin\theta d$ are considered.

$$F_\nu \approx -\frac{2\pi}{\alpha_\nu}\frac{\partial B_\nu^{med}(\nu,T)}{\partial x}\int_{-1}^{1}\mu^2\, d\mu = -\frac{2\pi}{\alpha_\nu}\frac{\partial B_\nu^{med}(\nu,T)}{\partial x}\frac{2}{3} = -\frac{4\pi}{3\alpha_\nu}\frac{\partial B_\nu^{med}(\nu,T)}{\partial x} \qquad (5.3.50)$$

thus,

$$F_\nu = -\frac{4\pi}{3\alpha_\nu}\frac{\partial B_\nu^{med}(T,x)}{\partial x} \qquad (5.3.51)$$

thus,

$$q_R = \int_0^\infty F_\nu\, d\nu = \int_0^\infty \left(-\frac{4\pi}{3\alpha_\nu}\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu = -\frac{4\pi}{3}\int_0^\infty \left(-\frac{1}{\alpha_\nu}\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu \qquad (5.3.52)$$

Rosseland mean absorption coefficient is defined as:

$$\frac{1}{\alpha_R} = \frac{\int_0^\infty \left(\frac{1}{\alpha_\nu}\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu}{\int_0^\infty \left(\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu} \tag{5.3.52b}$$

With Rosseland mean absorption coefficient, the equation (5.3.52) can also be expressed as:

$$q_R \approx -\frac{4\pi}{3}\frac{1}{\alpha_R}\int_0^\infty \left(\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu \tag{5.3.53}$$

Here, consider radiative thermal conductivity under dT/dx.

$$k_R \approx \frac{q\_R}{dT/dx} = \frac{4\pi}{3\alpha_R}\int_0^\infty \left(\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu \frac{dx}{dT} = \frac{4\pi}{3\alpha_R}\int_0^\infty \left(\frac{\partial B_\nu^{med}(T,x)}{\partial T}\right) d\nu \tag{5.3.54}$$

thus,

$$k_R = \frac{4\pi}{3\alpha_R}\frac{\partial}{\partial T}\left(\langle n^2\rangle \frac{\sigma}{\pi} T^4\right) = \frac{16}{3\alpha_R}\langle n^2\rangle \sigma T^3 \tag{5.3.55}$$

The equation (5.3.55') is mentioned in Clark's (1957):

$$k_R = \frac{16 n^2 \sigma T^3}{3\alpha_R} \tag{5.3.55'}$$

where k_R is the radiative thermal conductivity, n is the refractive index, T is the temperature, and α_R is the reciprocal of Rosseland mean absorption coefficient.

Temperature dependence

Wavenumber (or frequency, wavelength) distribution of Black-body function changes with temperature as shown in Figure 3.7. With increase temperature T, the absorbance α_R increases. Considering the equation (5.3.55'), k_R increases less than proportional to T^3. The mean emission shift to higher wavenumber (higher frequency, shorter wavelength) with increasing temperature. The absorption band shifts to lower wavenumber (lower frequency, longer wavelength) with temperature.

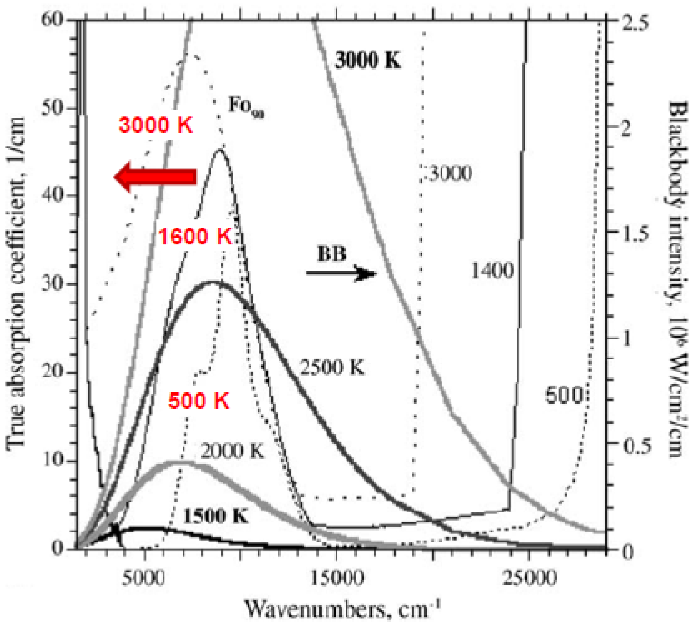

Fig.3.7 The absorption coefficient and thermal radiation of natural olivine with different temperatures reported in Hofmeister 2005.

Another thermal resistance

Minerals have a unique refractive index and optically anisotropic minerals can cause different refractive index. Therefore, on the grain boundary, matters with different refractive index neighbor on each other. Aside from above, scattering on a grain boundary can be another thermal resistance.

Consider photon free path due to the grain-boundary scattering:

$$\Lambda_{GB} = \frac{d}{R} \qquad (5.3.56)$$

where d is the grain size (diameter) and R is the interface reflectivity (fraction of reflected light). Here it is supposed that when wavenumber diverges to positive infinity, interface reflectivity R and refractive index n should approach towards 0 and 1, respectively.

$$\frac{1}{\Lambda_{tot}} = \frac{1}{\Lambda_{abs}} + \frac{1}{\Lambda_{GB}} = \alpha + \frac{R}{d} \qquad (5.3.56)$$

where Λ_{tot} is the total photon mean free path, Λ_{tabs} is the photon mean free path, Λ_{GB} is the photon free path due to the grain-boundary scattering, α is absorption coefficient, d is the grain size (diameter) and R is the interface reflectivity (fraction of reflected light). Radiative thermal conductivity of polycrystalline olivine (Hofmeister 2005). Grain-boundary scattering makes thermal conductivity more complicated than those shown in Figure3.7.

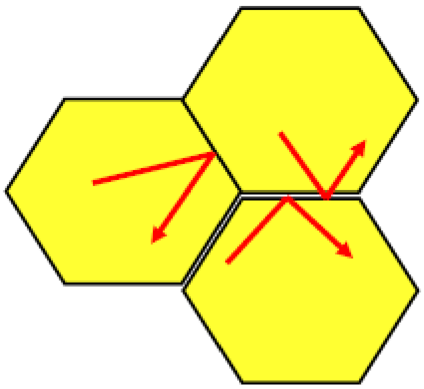

Fig. 3.8 A schematic image of grain boundary and reflection on that.

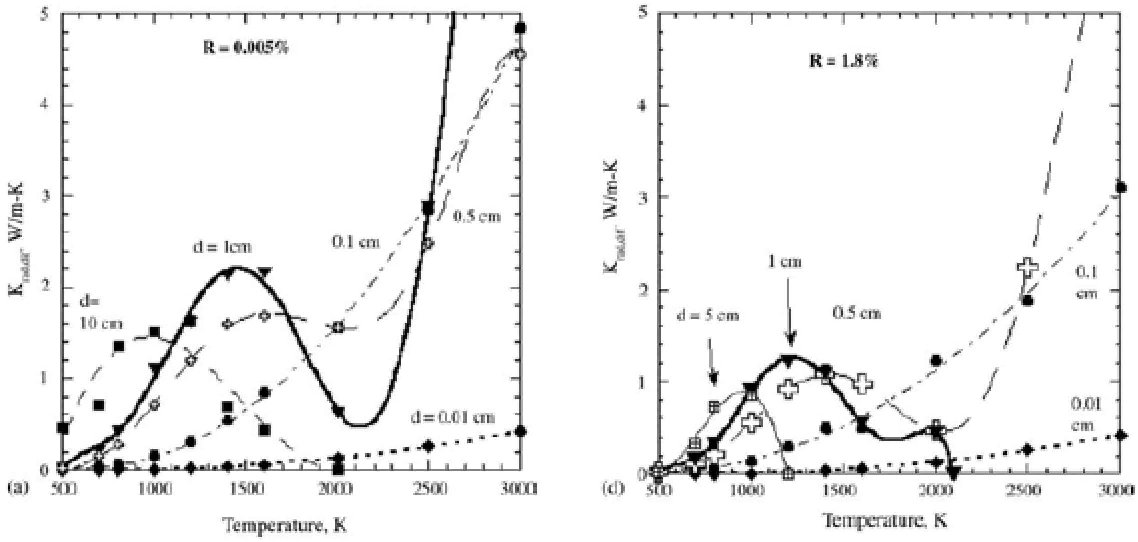

Fig.3.9 Radiative thermal conductivity of polycrystalline olivine (Hofmeister 2005). Grain-boundary scattering makes thermal conductivity complicated.