5. Heat transfer properties

3. Radiative thermal conductivity

3.1 Thermal radiation

A hot body emits electromagnetic waves, referred to as thermal radiation. The electromagnetic wave is generated by the thermal motion of charged particles in matter. All matters with a temperature T greater than 0 K emits thermal radiation. Another important point is that the internal energy of lattice vibration is converted to electromagnetic waves to be released to space. The thermal radiation has two basic features; it becomes stronger with increasing temperature and the color changes from red to white.

3.2 Term

We define four terms: radiance, radiant emittance, spectral radiance, and spectral energy density. Radiance is the radiant energy flux from a surface per unit solid angle and per unit area of the radiating surface. Radiant emittance is the radiant energy flux emitted by a surface per unit area. The difference between radiance and radiant emittance is included per unit solid angle or not. It means radiant emittance is an integration of radiance for all solid angles. Spectral radiance is radiance at a given frequency or wavelength. Spectral energy density is stored energy per unit volume at a given frequency or wavelength.

3.3 Specific intensity

A measure of energy flow is referred to as a specific intensity. The amount of energy $\mathrm{d}E_\nu$ transported through surface area dA is proportional to length time dt, frequency width dν, solid angle dω, and the projected unit surface area $\cos\theta$ dA. The proportionality factor is the specific intensity $I_\nu(\cos\theta)$ [$\mathrm{Jm^{-2}\ sr^{-1}\ Hz^{-1}\ s^{-1}}$]. The specific intensity is the function of the direction to the normal. It is defined as follows.

$$\mathrm{d}E_\nu = I_\nu(\cos\theta)\cos\theta\ \mathrm{dA\ d\omega\ d\nu\ dt} \qquad (5.3.1)$$

Intensity can convert from frequency to wavelength.

$$I_\lambda = \frac{c}{\lambda^2} I_\nu \qquad (5.3.2)$$

because $I_\lambda \mathrm{d}\lambda = I_\nu d\nu$, $\nu = \frac{c}{\lambda}$, then $d\nu = -\frac{c}{\lambda^2}\mathrm{d}\lambda$. c is the speed of the light, λ is the wavelength.

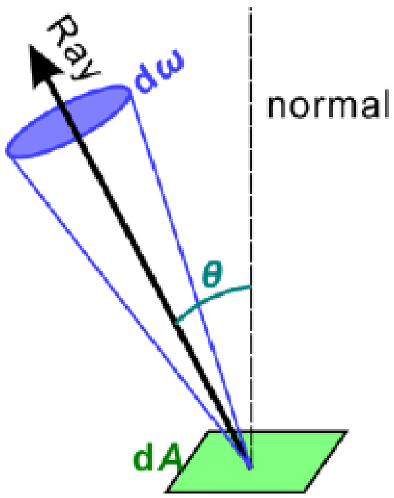

Fig. 1. Conceptional diagram of energy flow through surface area dA. Black arrow shows energy flow. Blue circle shows a solid angle dω. θ show the direction to the normal.

3.4 Radiative flux

We consider radiative flux. Energy flows through a surface element of A. $dE_\nu \propto I_\nu \cos\theta \, d\omega$ Flux, F_ν is defined as integrating the specific intensity over the half solid angle; half of semi-square 2π.

$$F_\nu = \int_{2\pi} I_\nu \cos\theta \, d\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta d\theta d\phi \quad (5.3.3)$$

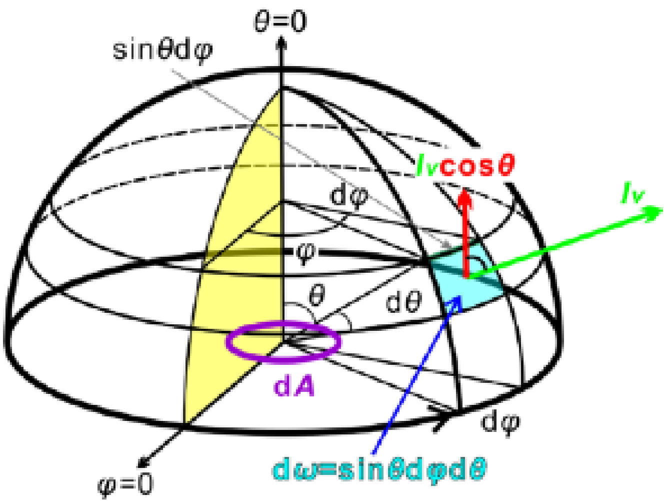

Fig. 2. Conceptional diagram of energy flow through surface area dA. Green arrow shows energy flow. θ, ϕ shows zenith and azimuth angle, respectively.

3.5 Planck's law of thermal radiation

Planck's law is an essential law of thermal radiation. It is the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature. The spectral density is

$$B_\nu(\nu, T) d\nu = \frac{2h\nu^3}{c^2} \frac{1}{exp\,(h\nu/k_B T) - 1} d\nu \quad (5.3.4)$$

where h is the Planck constant, k_B is the Boltzmann's constant, B_ν is the energy density. At relatively low temperature 5000K, energy density is very weak. With increasing temperature, energy density is much stronger, and peak frequency shifts to higher.

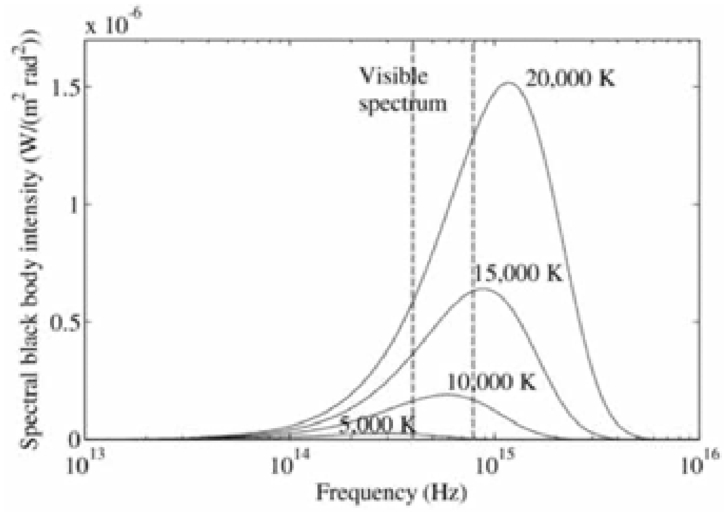

Fig. 3. Spectral black body intensity with frequency.

Plank's law can be expressed in many ways. One is the spectral radiance form. The unit is $Jm^{-2}sr^{-1}s^{-1}$. With frequency, the spectral radiance is given by

$$B_\nu(\nu, T)d\nu = \frac{2h\nu^3}{c^2}\frac{1}{exp\ (h\nu/k_BT) - 1}d\nu \tag{5.3.4}$$

With wavelength, the spectral radiance is given by

$$B_\lambda(\lambda, T)d\nu = \frac{2hc^2}{\lambda^5}\frac{1}{exp\ (h\nu/\lambda k_BT) - 1}d\nu \tag{5.3.5}$$

The other is the spectral energy density form. The unit is Jm^{-3}.

$$u(T) = \frac{4\pi}{c}B(T) \tag{5.3.6}$$

With frequency, the spectral energy density is given by

$$B_\nu(\nu, T)d\nu = \frac{8\pi h\nu^3}{c^2}\frac{1}{exp\ (h\nu/k_BT) - 1}d\nu \tag{5.3.7}$$

With wavelength, the spectral energy density is given by

$$B_\lambda(\lambda, T)d\nu = \frac{8\pi hc^2}{\lambda^5}\frac{1}{exp\ (h\nu/\lambda k_BT) - 1}d\nu \tag{5.3.8}$$

3.6 Derivation of Planck's law

We derive Plank's law in this section. We consider a black body and an empty black-body container. A black body is an object capable of emitting and absorbing all wavelengths of light completely. Empty black-body container maintained at a constant temperature. As a result, radiation is absorbed and emitted on the wall. In the equilibrium condition, the thermal radiation exists the stationary waves. If we consider the dimension of container L, the wavelength and frequency of the stational wave are expressed as follows.

$$\lambda = 2L, \frac{2L}{2}, \frac{2L}{3}, \dots, \frac{2L}{n}, \dots \tag{5.3.9}$$

$\nu = \frac{c}{\lambda_n} = \frac{nc}{2L}$	(5.3.10)

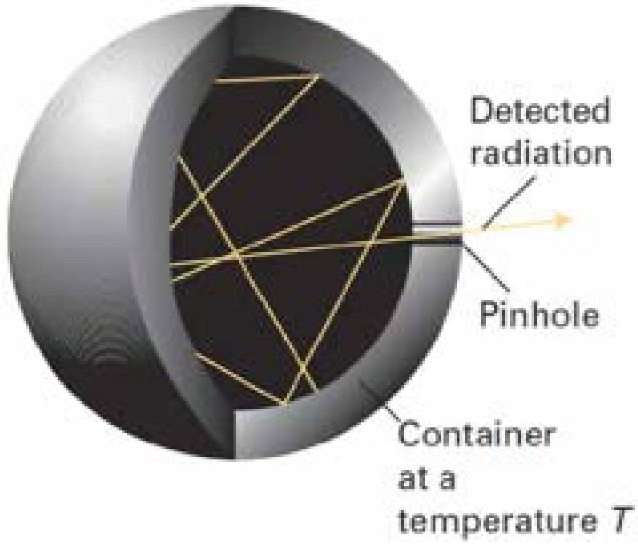

Fig. 4. Conceptional diagram of a black body with an empty container.

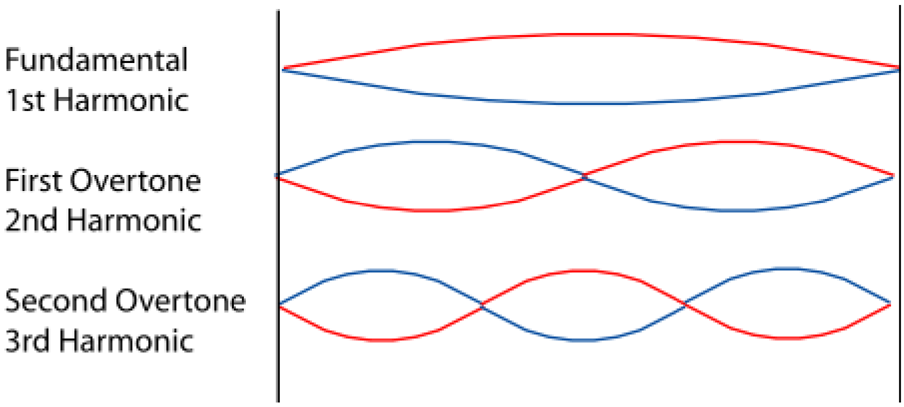

Fig. 5. Conceptional diagram of a stationary wave.

We observe the leaking-out of the light through a pinhole. A set of stationary waves of thermal radiation is expressed as follows.

$E_x = E_{x0} \sin \omega t \cos(n_x \pi x/L) \sin(n_y \pi x/L) \sin(n_z \pi x/L)$	(5.3.11a)
$E_y = E_{y0} \sin \omega t \sin(n_x \pi x/L) \cos(n_y \pi x/L) \sin(n_z \pi x/L)$	(5.3.11b)
$E_z = E_{z0} \sin \omega t \cos(n_x \pi x/L) \sin(n_y \pi x/L) \cos(n_z \pi x/L)$	(5.3.11c)

where E_x, E_y, E_z is the amplitude of the electric fields propagating in the x, y, and z direction. Substituting (5.3.11a) -(5.3.11c) into the wave equation: $\frac{\partial^2 E}{\partial t^2} = c^2 \nabla^2 E$

$$\omega^2\mathbf{E} = c^2\frac{(n_x^2+n_y^2+n_z^2)}{L^2}\boldsymbol{E}$$ $$\omega^2 = \frac{\pi^2c^2}{L^2}(n_x^2+n_y^2+n_z^2)$$ $$\nu = \frac{c}{2L}(n_x^2+n_y^2+n_z^2)^{\frac{1}{2}}$$	(5.3.12)

Then, we define

$$n^2 \equiv n_x^2+n_y^2+n_z^2$$	(5.3.13)

Then, (5.3.12) is

$$\nu = \frac{c}{2L}(n_x^2+n_y^2+n_z^2)^{\frac{1}{2}} = \frac{c}{2L}n$$	(5.3.14)

Next, we consider the density of modes between n and $n+dn$, $D(n)dn$. There are two directions of vibration (polarization) of the electric field, and we multiply the factor 2. $n = (n_x^2+n_y^2+n_z^2)^{\frac{1}{2}}$ means spherical distribution in an n-space: $4\pi n^2 dn$ mode between n and $n+dn$. Because of the stationary wave, $n_x, n_y, n_z > 0$. Therefore, only $\left(\frac{1}{2}\right)^2$ of $4\pi n^2$ sphere should be value. The density mode between n and $n+dn$ is

$$D(n)dn = 2\times\frac{1}{8}\times 4\pi n^2 dn = \pi n^2 dn$$	(5.3.15)

The density of states of photons between ν and $\nu+d\nu$, $g(\nu)d\nu$

$$g(\nu)d\nu = \pi\left(\frac{2L}{c}\nu\right)^2\frac{2L}{c}d\nu = \frac{8\pi L^3}{c^3}\nu^2 d\nu$$	(5.3.16)

Substituting the equation (5.3.9) $n = \frac{2L}{c}\nu$.

From the Planck distribution, the average energy at a given frequency, ν, at T is given by Eq. (3.5.7)

$$\langle\varepsilon(\nu)\rangle = \frac{h\nu}{exp(h\nu/k_BT)-1}$$	(3.5.7)

By combining Eq. (3.5.7) and (5.3.12) $g(\nu)d\nu = \frac{8\pi L^3}{c^3}\nu^2 d\nu$, we have the spectral E in the volume L^3 at a given ν

$$U_\nu(\nu,T)d\nu = \frac{h\nu}{exp(h\nu/k_BT)-1}\frac{8\pi L^3}{c^3}\nu^2 d\nu = \frac{8\pi L^3 h\nu^3}{c^3}\frac{1}{exp(h\nu/k_BT)-1}d\nu$$	(5.3.17)

The spectral energy density is obtained by normalizing (5.3.17) by $V = L^3$

$$u_\nu(\nu,T)d\nu = \frac{U_\nu(\nu,T)d\nu}{L^3} = \frac{8\pi h\nu^3}{c^3}\frac{1}{exp(h\nu/k_BT)-1}d\nu$$	(3.5.7)

This is Planck's law of thermal radiation by spectral energy density.

We derive Planck's law of spectral radiance. The spectral radiance is defined as the energy emitted per unit solid angle and time. When calculating it, we consider two things. One is the unit direction normalized by the whole solid angle 4π. The other is the unit time, which is the length that the light has swept for a unit time $c\Delta t$. The spectral radiance is

$$B_\nu(\nu,\mathrm{T})\mathrm{d}\nu = \frac{c}{4\pi}u_\nu(\nu,T)d\nu = \frac{c}{4\pi}\frac{8\pi h\nu^3}{c^3}\frac{1}{exp\ (h\nu/k_BT)-1}d\nu$$ $$= \frac{2h\nu^3}{c^2}\frac{1}{exp\ (h\nu/k_BT)-1}d\nu$$	(5.3.4)

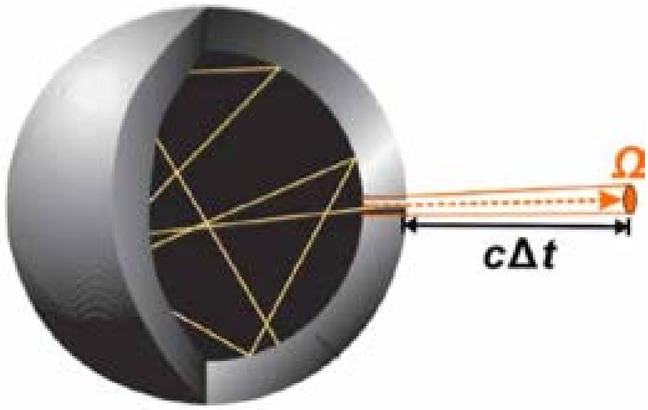

Fig. 6. Conceptional diagram of a black body with an empty container. Through the pinhole, energy is emitted.

3.7 Derivation of Stefan-Boltzmann law

We derive Stefan-Boltzmann law. The total E radiated per unit surface area of a black body across all ν per unit t is directly proportional to T^4. Radiance L

$$L = \frac{\sigma}{\pi} T^4$$	(5.3.18)

Radiant emittance j^*

$$j^* = \sigma T^4$$	(5.3.19)

Energy density U

$$U = \frac{4\sigma}{c} T^4$$	(5.3.20)

Stefan-Boltzmann constant σ

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67 \ldots \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$$	(5.3.21)

Integration of Eq. (5.3.3) over all ν

$$L = \int_0^\infty B_\nu(\nu, \mathrm{T})\mathrm{d}\nu = \int_0^\infty \frac{2h\nu^3}{c^2} \frac{1}{exp\,(h\nu/k_BT) - 1} \mathrm{d}\nu = \frac{2h}{c^2}\left(\frac{k_BT}{h}\right)^4 \int_0^\infty \frac{\left(\frac{h\nu}{k_BT}\right)^3}{e^{\frac{h\nu}{k_BT}} - 1} \frac{h}{k_BT} \mathrm{d}\nu$$

$$L = \frac{2h}{c^2}\left(\frac{k_BT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} \mathrm{d}x$$	(5.3.22)

where we define $x = \frac{h\nu}{k_BT}$.

$$L = \frac{2h}{c^2}\left(\frac{k_BT}{h}\right)^4 \frac{\pi^4}{15} = \frac{2\pi^4 k_B^4}{15c^2h^3} T^4 = \frac{\sigma}{\pi} T^4$$	(5.3.18)

Then we use this formula

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$	(5.3.23)

Radiant emittance is the integration of radiance L over all radiation in the direction normal to the surface.

$$j^* = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L \cos\theta \sin\theta d\phi d\theta = \frac{\sigma}{\pi} T^4 \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta d\theta \int_0^{2\pi} d\phi = \frac{\sigma}{\pi} T^4 \int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{2} d\theta\ 2\pi = \sigma \left[\frac{1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}} T^4 = \sigma T^4 \tag{5.3.19}$$

3.7 Radiation laws in a transparent medium

We confirm the specific argument for this field. The velocity of light in a medium q is slower than in vacuum: $q < c$. The refractive index n

$$n = \frac{c}{q} \tag{5.3.24}$$

Planck's law (5.3.4) in the medium,

$$B_\nu^{med}(\nu, T) = \frac{2h\nu^3}{q^2} \frac{1}{exp\,(h\nu/k_B T) - 1} = \left(\frac{c}{q}\right)^2 \frac{2h}{c^2} \frac{\nu^3}{exp\,(h\nu/k_B T) - 1} = n^2 B_\nu(\nu, T) \tag{5.3.25}$$

Stefan-Boltzmann law (5.3.18) in the medium,

$$L = \int_0^\infty n^2 B_\nu^{med}(\nu, T) d\nu = \langle n^2 \rangle \frac{\sigma}{m} T^4 \tag{5.3.26}$$

$$\langle n^2 \rangle = \frac{\int_0^\infty n^2 B_\nu(\nu, T) d\nu}{\int_0^\infty B_\nu(\nu, T) d\nu} \tag{5.3.27}$$

The radiated energy in a medium is higher than vacuum, proportionally to $\langle n^2 \rangle$.

3.8 Thermal resistance of radiative heat transfer

We consider the thermal resistance of radiative heat transfer. If there is a perfectly transparent material, it is no thermal conductivity by thermal radiation. If the medium absorbs light to some degree, energy is transferred from one part of the body to another. It causes thermal resistance. Therefore, the optical absorption of light is essential for radiative heat transfer.

3.9 Absorbance & absorption coefficient

We consider several terms of absorption. The intensity of light passing through a semi-transparent body is exponentially attenuated,

$$I_{tr}(s) = I_0 \exp(-\alpha s) \approx I_0(1 - \alpha s) \tag{5.3.28}$$

where, I_{tr} means the intensity of transmitted light and I_0 means the intensity of incident light and α means absorption coefficient [m^{-1}] and s is path.

$$dI = I\ d \exp(-\alpha s) \approx I\ d(1 - \alpha s) = -\alpha I ds \tag{5.3.29}$$

Absorbance

$$A = \ln\left(\frac{I_0}{I_{tr}}\right) = \ln\left(\frac{I_0}{I_0 \exp(-\alpha s)}\right) = \alpha s \tag{5.3.30}$$

In many cases, absorbance uses not the natural logarithm (ln) but the common logarithm ($\log_{10}$).

3.10 Absorptivity

We consider absorptivity. The intensity of the light absorbed by the body,

$I_{abs} = I_0 - I_{tr} = I_0 - I_0 \exp(-\alpha d) = I_0(1 - \exp(-\alpha s))$	(5.3.31)

Absorptivity ξ is the fraction of total incident radiation which is absorbed by a body.

$\xi = \frac{I_{abs}}{I_0} = \frac{I_0 - I_{tr}}{I_0}$	(5.3.32)
$\xi = \frac{I_0 - I_0 \exp(-\alpha d)}{I_0} = 1 - \exp(-\alpha s)$	(5.3.33)

3.11 Photon mean free path

We consider photon mean free path. The photon mean free path, Λ_{pht} is the average length of transmission of the light.

$\Lambda_{pht} = \frac{\int_0^\infty sI_0 \exp(-\alpha s)\, ds}{\int_0^\infty I_0 \exp(-\alpha s)\, ds} = \frac{\frac{1}{\alpha^2}\int_0^\infty (-\alpha)x \exp(-\alpha s)\,(-\alpha)ds}{-\frac{1}{\alpha}\int_0^\infty I_0 \exp(-\alpha s)\,(-\alpha)ds} = -\frac{1}{\alpha}\frac{\int_0^\infty ze^{-z}dz}{\int_0^\infty e^{-z}dz} = \frac{1}{\alpha}$	(5.3.34)

Using the following equations.

$$\int_0^\infty e^{-z}dz = [-e^{-z}]_0^\infty = -e^{-\infty} + 1 = 1$$

$$\int_0^\infty ze^{-z}dz = \int_0^\infty z\,(-e^{-z})'dz = [x(-e^{-z})]_0^\infty - \int_0^\infty (z)'(-e^{-z})dz = 0 + 1 = 1$$

The photon mean free path means the reciprocal absorption coefficient.

$I_{tr}(s) = I_0 \exp(-\frac{s}{\Lambda_{pht}})$	(5.3.35)

3.12 The equation of radiative heat transfer

We consider the equation of the radiative heat transfer. Energy added by emission within a volume $dV = dAds$

$dE_\nu^{em} = \epsilon_\nu dVd\omega d\nu dt = \epsilon_\nu dAdsd\omega d\nu dt$	(5.3.36)

where ϵ_ν means emission coefficient [$Jm^{-3}sr^{-1}Hz^{-1}s^{-1}$]

From (5.3.23)

$dE_\nu^{abs} = dI_\nu^{abs} dAd\omega d\nu dt = -\alpha_\nu I_\nu dAd\omega d\nu dtds$	(5.3.37)

The net change in specific intensity

$dI_\nu = -\alpha_\nu I_\nu ds + \epsilon_\nu ds$	(5.3.38)
$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + \epsilon_\nu$	(5.3.39)

The equation describes the flow of radiation through the matter.

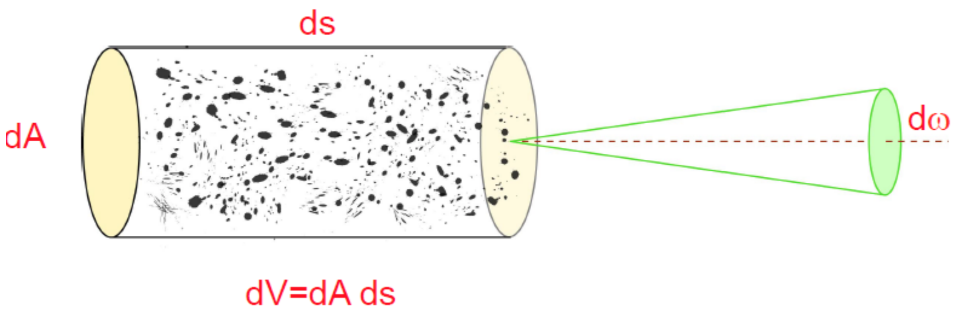

Fig. 7. Conceptional diagram of a matter with a volume dV that emits energy.

We consider plane-parallel symmetry. The path of x-direction in the temperature gradient

$dx = \cos\theta\, ds = \mu ds$	(5.3.40)
$\mu = \cos\theta$	(5.3.41)
$\frac{d}{ds} = \mu \frac{d}{dx}$	(5.3.42)

Therefore, the equation of radiative heat transfer (5.3.39)

$\mu \frac{dI_\nu(\mu, x)}{ds} = -\alpha_\nu(x) I_\nu(\mu, x) + \epsilon_\nu(x)$	(5.3.43)

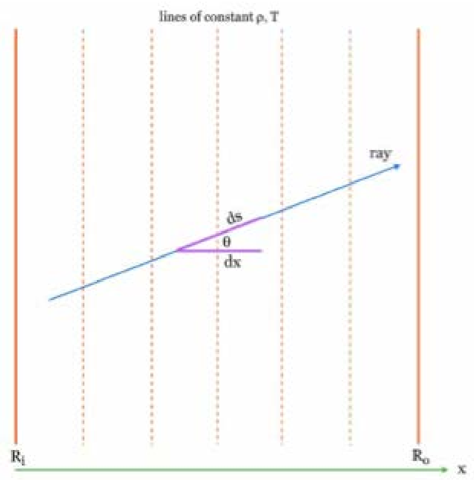

Fig. 8. Conceptional diagram of energy flow within plane-parallel matter.

Then, we normalized by α_ν

$\frac{\mu}{\alpha_\nu} \frac{dI_\nu(\mu, x)}{ds} = -I_\nu(\mu, x) + \frac{\epsilon_\nu(x)}{\alpha_\nu(x)} = S_\nu(x) - I_\nu(\mu, x)$	(5.3.44)

where source function,

$S_\nu(x) = \frac{\epsilon_\nu(x)}{\alpha_\nu(x)}$	(5.3.45)

Next, we consider local thermal equilibrium (LTE). It means a balance of emission and absorption (Kirchhoff's law). Emissivity

$\varepsilon_\nu(x) = \epsilon_\nu(x)/B_\nu(x)$	(5.3.46)
$\xi_\nu(x) \approx \alpha_\nu(x)$	(5.3.47)
$S_\nu(x) = \dfrac{\epsilon_\nu(x)}{\alpha_\nu(x)} = \dfrac{\varepsilon_\nu(x) B_\nu(x)}{\alpha_\nu(x)} \approx B_\nu(x)$	(5.3.48)

3.13 Radiative thermal conductivity

We consider the radiative thermal conductivity under $\frac{dT}{dx}$. When we consider plane-parallel symmetry (5.3.44) and local thermal equilibrium (5.3.48), the spatial variation of the local intensity

$I_\nu(\mu, x) = B_\nu^{med}(x) - \dfrac{\mu}{\alpha_\nu}\dfrac{dI_\nu(\mu, x)}{dx} \approx B_\nu^{med}(x) - \dfrac{\mu}{\alpha_\nu}\dfrac{\partial B_\nu^{med}(x)}{\partial x}$	(5.3.49)

The energy flux at ν integrated over all direction from (5.3.3)

$F_\nu = \int_{4\pi} I_\nu \mu d\Omega = \int_{4\pi} B_\nu^{med}(T, x) d\Omega - \int_0^{2\pi} d\phi \int_0^{\pi} \dfrac{\mu}{\alpha_\nu}\dfrac{\partial B_\nu^{med}(x)}{\partial x} \cos\theta \sin\theta d\theta$ $F_\nu \approx -\dfrac{2\pi}{\alpha_\nu}\dfrac{\partial B_\nu^{med}(x)}{\partial x}\int_{-1}^{1} \mu^2 d\mu = -\dfrac{4\pi}{3\alpha_\nu}\dfrac{\partial B_\nu^{med}(x)}{\partial x}$	(5.3.50)

Using $\int_{4\pi} B_\nu^{med}(T, x) d\Omega \approx 0$, $\mu = \cos\theta$, $d\mu = -\sin\theta$. From (5.3.50) the energy flux at ν is

$F_\nu = -\dfrac{4\pi}{3\alpha_\nu}\dfrac{\partial B_\nu^{med}(x)}{\partial x}$	(5.3.51)

Heat influx is the integrating the energy flux at ν over all ν

$q_{rad} = \int_0^\infty F_\nu d\nu = \int_0^\infty \left(-\dfrac{4\pi}{3\alpha_\nu}\dfrac{\partial B_\nu^{med}(T, x)}{\partial x}\right) d\nu$ $= -\dfrac{4\pi}{3}\int_0^\infty \left(\dfrac{1}{\alpha_\nu}\dfrac{\partial B_\nu^{med}(T, x)}{\partial x}\right) d\nu$	(5.3.52)

When we use the Rosseland mean absorption coefficient $\frac{1}{\alpha_R} = \frac{\int_0^\infty \left(\frac{1}{\alpha_\nu}\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu}{\int_0^\infty \left(\frac{\partial B_\nu^{med}(T,x)}{\partial x}\right) d\nu}$,

$q_{rad} \approx -\dfrac{4\pi}{3\alpha_\nu}\dfrac{1}{\alpha_R}\int_0^\infty \left(\dfrac{\partial B_\nu^{med}(T, x)}{\partial x}\right) d\nu$	(5.3.53)

Radiative thermal conductivity,

$k_R = -\dfrac{q_{rad}}{dT/dz} = \dfrac{4\pi}{3\alpha_\nu}\int_0^\infty \left(\dfrac{\partial B_\nu^{med}(T, x)}{\partial x}\right) d\nu \dfrac{dz}{dT} = \dfrac{4\pi}{3\alpha_\nu}\int_0^\infty \left(\dfrac{\partial B_\nu^{med}(T, x)}{\partial T}\right) d\nu$	(5.3.54)
$k_R \approx \dfrac{4\pi}{3\alpha_R}\dfrac{\partial}{\partial T}\int_0^\infty B_\nu^{med}(T, x)\, d\nu$ $k_R = \dfrac{4\pi}{3\alpha_R}\dfrac{\partial}{\partial T}\left(\langle n^2\rangle \dfrac{\sigma}{\pi} T^4\right) = \dfrac{16}{3\alpha_R}\langle n^2\rangle \sigma T^3$	(5.3.55)

It is known as Clark's (1957) formula.

$$k_R = \frac{16n^2\sigma T^3}{3\alpha_R}$$	(5.3.55’)

3.14 Temperature dependence

We consider temperature dependence of thermal conductivity. With increasing temperature, the absorbance α_R increases. It causes that the radiative thermal conductivity increases less than proportional to temperature (5.3.55’).

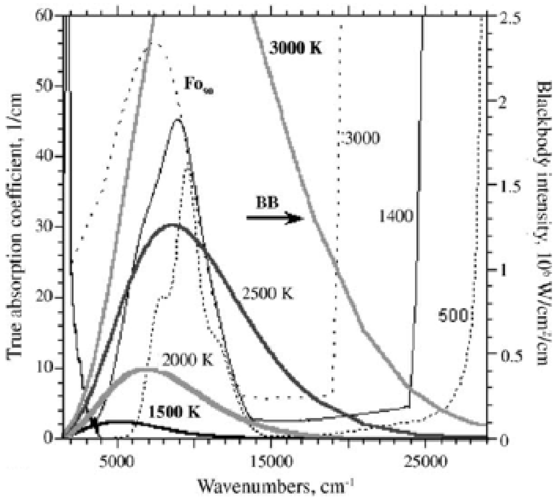

Fig. 9. Planck’s thermal radiation with a function of wavelength.

3.15 Another thermal resistance

Another thermal resistance is the scattering in the grain boundary. If we assume interface reflectivity R and grain size d, the photon free path due to the grain-boundary scattering is

$$\Lambda_{GB} = \frac{d}{R}$$	(5.3.56)

If R=0, the photon free path is infinite. It means transparent. If R=1, the photon free path is related to grain size. Therefore, the grain size is important for radiative heat transfer. The total photon mean free path Λ_{tot}

$$\frac{1}{\Lambda_{tot}} = \frac{1}{\Lambda_{abs}} + \frac{1}{\Lambda_{GB}} = \alpha + \frac{R}{d}$$	(5.3.57)

where $\Lambda_{abs} = 1/\alpha$ (5.3.4).

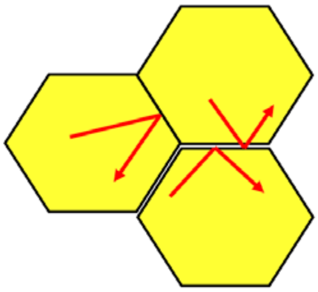

Fig. 10. Conceptional diagram of scattering in the grain boundary.