5. Heat transfer properties

Importance of heat transfer in geophysics.

It is not enough to say that the Earth interior is essentially hot. It is not just “hot”, but the temperature varies from region to region. High temperatures lead to changes in density due to the thermal expansion and generally the higher temperatures, the lower the density. In addition, heterogeneous temperature distribution of temperatures inside the Earth leads to regional buoyancy, which gives rise to dynamic motion of the internal masses of the Earth (convection). He dynamic motion include: slab subduction, plume ascent, dynamo in the outer core. To consider these processes, knowledge of the heat transfer properties in the Earth’s constituents is required.

1. Physics of heat transfer

1.1 Classification of heat transfer mechanism

There are three heat transfer mechanisms.

1. Conductive heat transfer.

If two bodies contact, the heat flows from the high-T body to low-T body. In this mechanism, energy is transferred due to atomic vibration. The higher the temperature of the body, the more the atoms vibrate. Correspondingly, stronger vibrations of atoms in the high-temperature one excite vibrations in the low-temperature one, due to which the temperature of the last one. The carrier particle of heat in the conductive mechanism of heat transfer is a phonon.

The conductive mechanism of heat transfer describes the Fourier’s law.

$$q = -k\left(\frac{dT}{dx}\right) \tag{5.1.1}$$

where q is the heat flow, transferred heat per unit area and per unit time, the proportional parameter k is the thermal conductivity, T is the temperature, x is the coordinate and dT/dx is the temperature gradient.

Thus, the heat flow is proportional to the temperature gradient and, importantly, does not depend on the absolute value of the temperature.

2. Radiative heat transfer.

A body with a high temperature emits light, otherwise it is called thermal radiation. The intensity of radiative heat transfer is described by the Stefan-Boltzmann’s law, according to which the radiation power is proportional to the absolute value of temperature to the T^4. According to this heat transfer mechanism, energy is transferred with emitted light. In this case, the energy carrier particle is a photon.

3. Convective heat transfer.

Heat is transferred by movement of high-T body. If completely convective, the temperature distribution is adiabatic.

1.2 Heat transfer in space

Before considering heat transfer in a material, we consider radiative heat transfer in “space” or vacuum. In such case, heat transfer is possible only by thermal radiation. A high-temperature body radiates energy as it tends to thermal equilibrium with vacuum, the temperature of which is zero.

The radiation of electromagnetic waves in vacuum is described by the Stefan-Boltzmann’s law:

$$\varepsilon = \sigma T^4 \tag{5.1.2}$$

where ε is the emitted energy, σ is the Stefan-Boltzmann constant σ = 5.670367×10^{-8} kg·s^{-3}·K^{-4}.

Thus, the total energy of electromagnetic wave emitted from a black body is proportional to the fourth power of temperature.

Let's consider heat transfer between surface A and B in parallel in a space. Let T_A and T_B be the temperature of surfaces A and B respectively. Energy emmited from A and B per unit area per time is: $q_A = \sigma T_A^4$ and $q_B = \sigma T_B^4$. Net heat flow from A to B is: $q = q_A - q_B = \sigma T_A^4 - \sigma T_B^4$. If $T_A \approx T_B$, then

$$q \approx \sigma \Delta T \cdot T_{AB}^3, \qquad \Delta T = T_A - T_B \tag{5.1.3}$$

Thus, the heat flow is proportional to the T-difference and also T^3. Note that the heat flow in vacuum is independent from the distance between the two surfaces.

Comparing with conductive hear transfer, which is described by the Fourier law: $q = -k\left(\frac{dT}{dx}\right) = -k\frac{\Delta T}{\Delta x}$, it can be seen that the heat flow is also proportional to the temperature difference, but also inversely proportional to the distance between the surfaces. Accordingly, there is a mechanism that causes an inverse dependence of conductive heat transfer on distance.

1.3 Thermal diffusion

Let's consider a thin plate with thickness δ_x and area S (fig. 1).

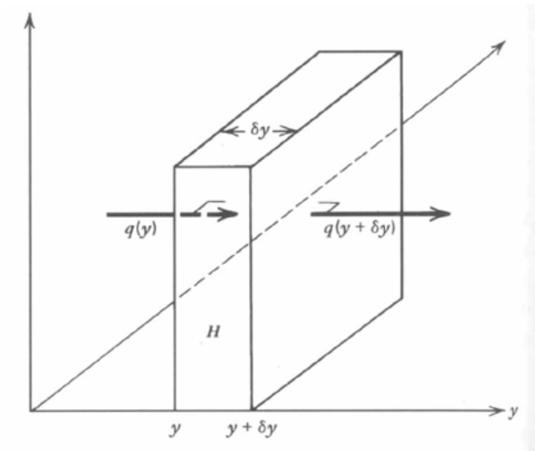

Fig. 1. The illustration of the heat flow q (the direction is indicated by bold arrows) through a thin plate with thickness δ_y and height H. The heat flow is directed along the y axis, the y axis is replaced by the x axis.

Let $q(x)$ be the heat flow from the left side into the plate. Let $q(x+\delta x)$ be heat flow from the plate to the right side. Increase in thermal energy in the plate per unit time is: $Q = S\{q(x) - q(x+\delta x)\}$. Thus,

$$Q = -S\left(\frac{\partial q}{\partial x}\right)_t \delta_x = -\delta_x S\left(\frac{\partial\left[-k\left(\frac{\partial T}{\partial x}\right)_t\right]}{\partial x}\right) = k\delta_x S\left(\frac{\partial T^2}{\partial x^2}\right)_t \tag{5.1.4}$$

Let consider the case, when T increasing rate at a position, $\left(\frac{\partial T}{\partial t}\right)_x$

$\left(\frac{\partial T}{\partial t}\right)_x = \frac{Q}{\rho C\delta_x S} = \frac{k\delta_x S}{\rho C\delta_x S}\left(\frac{\partial T^2}{\partial x^2}\right)_t$, where $\delta_x S$ is the volume of the plate, C is the specific heat per weight and ρC is the specific heat per volume. Therefore, the thermal diffusion equation is:

$$\left(\frac{\partial T}{\partial t}\right)_x = \kappa\left(\frac{\partial T^2}{\partial x^2}\right)_t \tag{5.1.5}$$

where proportional parameter $\kappa = \frac{k}{\rho C}$ is the thermal diffusivity.

1.4 Heat flow and temperature change with time.

Homogenous high temperature.

Let’s consider the case when the body temperature is constant (Fig. 2). The temperature distribution in this case will be uniform. The heat flow will be equal to zero, since it depends on the temperature gradient, which is equal to zero. There will also be no change in temperature over time, as follows from the thermal diffusion equation.

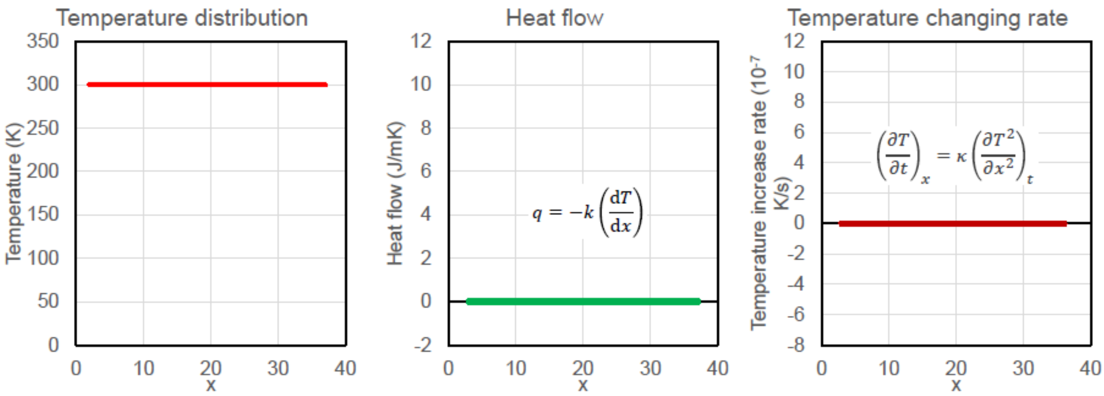

Fig. 2. The diagrams of the temperature distribution, heat flow and temperature changing rate for the homogenous high temperature.

Linear temperature gradient.

Let’s consider the next case, where the temperature distribution is linear (Fig. 3). Suppose the temperature decreases linearly with increasing x. In this case, the heat flow will not be equal to zero, since the temperature gradient is not equal to zero. In addition, it will be positive, since in this case the gradient is negative, but it is compensated by the minus on the right side of the Fourier equation. The change in temperature with time in this case is zero, since the second derivative of the linear function is zero.

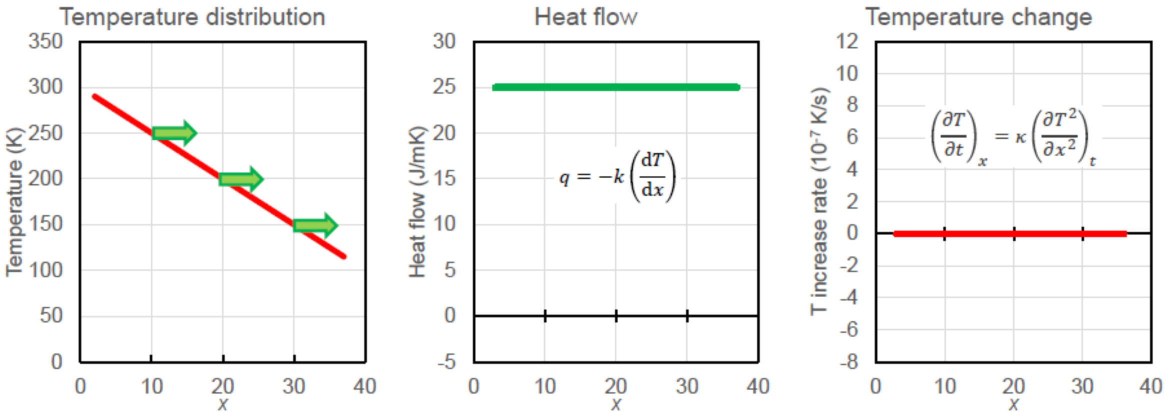

Fig. 3. The diagrams of the temperature distribution, heat flow and temperature changing rate for the linear temperature gradient.

Parabolic temperature distribution.

If temperature distribution is parabolic, it is non-linear (Fig. 4). In accordance with the temperature distribution function, in this particular case, the heat flow will decrease linearly, in accordance with the temperature gradient, but with the opposite sign. The change in temperature over time is non-zero and in this case is constant and does not depend on position.

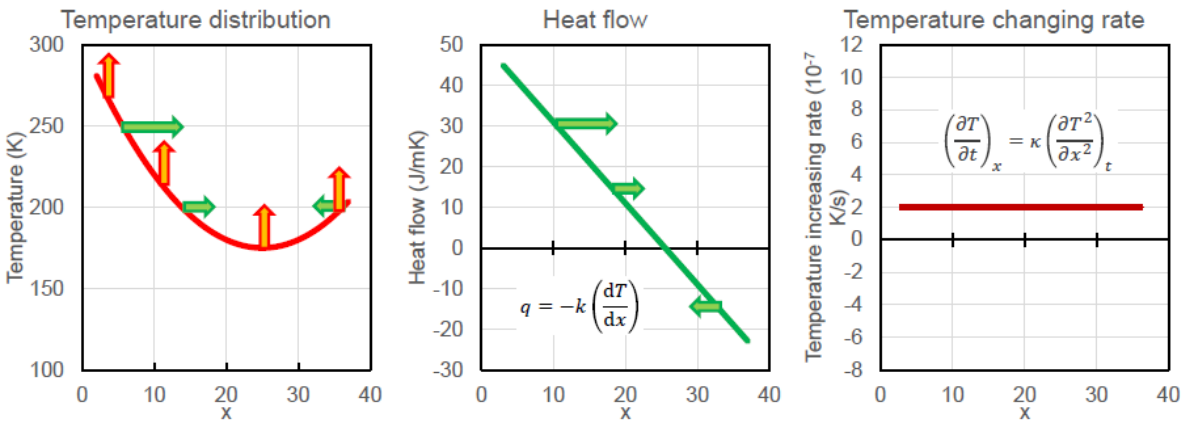

Fig. 4. The diagrams of the temperature distribution, heat flow and temperature changing rate for the parabolic temperature distribution.

Varying curvature with position.

Consider the case when the temperature distribution is not parabolic, but, for example, cubic. In this case, the heat flow decreases, and its magnitude increases with increasing temperature gradient. In this case, the temperature increases linearly with time.

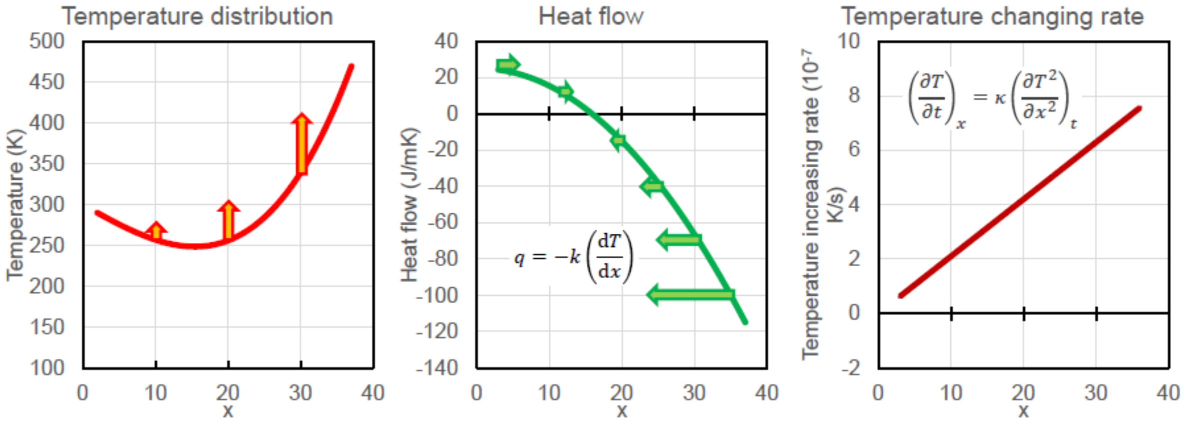

Fig. 4. The diagrams of the temperature distribution, heat flow and temperature changing rate for the varying curvature with position.