Mineral Physics I
Chapter 5. Heat transfer properties

TOMOO KATSURA

BAYERISCHES GEOINSTITUT, UNIVERSITY OF BAYREUTH, GERMANY

Importance of heat transfer in geophysics

- The Earth's interior: essentially "hot"
- Not just "hot", but the temperature varies from region to region
- High temperatures → Density variation due to the thermal expansion
  - Usually higher temperature → lower density
- Heterogeneous temperature distribution → regional buoyancy → dynamic motion within the Earth
  - Slab subduction, plume ascent, dynamo in the outer core
- In order to consider these problems, we need knowledge of the heat transfer properties of the Earth's constituents

Mineral Physics I
Chapter 5. Heat transfer properties
Section 1. Physics of heat transfer

TOMOO KATSURA

BAYERISCHES GEOINSTITUT, UNIVERSITY OF BAYREUTH, GERMANY

Classification of heat transfer mechanisms -1

1. **Conductive heat transfer**

Ø If two bodies contact, the heat flows from the high-T body to low-T body

Ø Energy transfer by propagation of atomic vibration

ü Atomic vibration: vigorous in HT, feeble in LT

ü Carrier particle: phonon

Ø Fourier's law

ü $q = -k(\mathrm{d}T/\mathrm{d}x)$ (5.1.1)

§ q: heat flow, transferred heat per unit area and per unit time

§ k: thermal conductivity

§ Independent of the absolute temperature

Classification of heat transfer mechanisms -2

2. **Radiative heat transfer**
   - Ø High-T body: emits light
     - ü thermal radiation
     - ü Stephan-Boltzmann's law
       - § Emission proportional to T^4
   - Ø Energy transfer through emitted light
   - Ø Carrier particle: photon

3. **Convective heat transfer**
   - Ø Heat is transferred by movement of high-T body
   - Ø If completely convective, the temperature distribution is adiabatic
     - ü Section 5 in Chapter 1: adiabat

Heat transfer in space -1

- Before considering heat transfer in a material, we consider radiative heat transfer in “space” or vacuum
- Thermal radiation
  - Electromagnetic wave is emitted from a HT-body
    - because the body and space try to become in thermal equilibrium
  - Stefan-Boltzmann law of radiation
    - $\varepsilon = \sigma T^4$ (5.1.2)
      - σ : Stefan-Boltzmann constant
        - $\sigma = 5.670367 \times 10^{-8}$ kg s^{-3} K^{-4}
    - Total energy of electromagnetic wave emitted from a black body is proportional to the fourth power of temperature

Heat transfer in space -2

- Heat transfer between surface A and B in parallel in a space
  - T_{A}: T of Surface A, T_{B}: T of Surface B
- Energy emitted from A and B per unit area per time: $q_{\mathrm{A}} = \sigma {T_{\mathrm{A}}}^4$, $q_{\mathrm{B}} = \sigma {T_{\mathrm{B}}}^4$
- Net heat flow from A to B: $q = q_{\mathrm{A}} - q_{\mathrm{B}} = \sigma {T_{\mathrm{A}}}^4 - \sigma {T_{\mathrm{B}}}^4$
- If $T_{\mathrm{A}} \approx T_{\mathrm{B}}$, $q \approx \sigma \Delta T \cdot T_{\mathrm{AB}}^3$, $\Delta T = T_{\mathrm{A}} - T_{\mathrm{B}}$ (5.1.3)
- Heat flow: proportional to the *T*-difference and also T^3.
- Independent from the distance between the two surfaces
  - Conductive heat transfer $q = -k \left(\frac{\mathrm{d}T}{\mathrm{d}x}\right) \approx -k \left(\frac{\Delta T}{\Delta x}\right)$
    - Proportional to *T*-difference AND inversely to the distance:
      - There is a special mechanism to cause the proportionality to distance in the conductive heat transfer

Thermal diffusion -1

a S

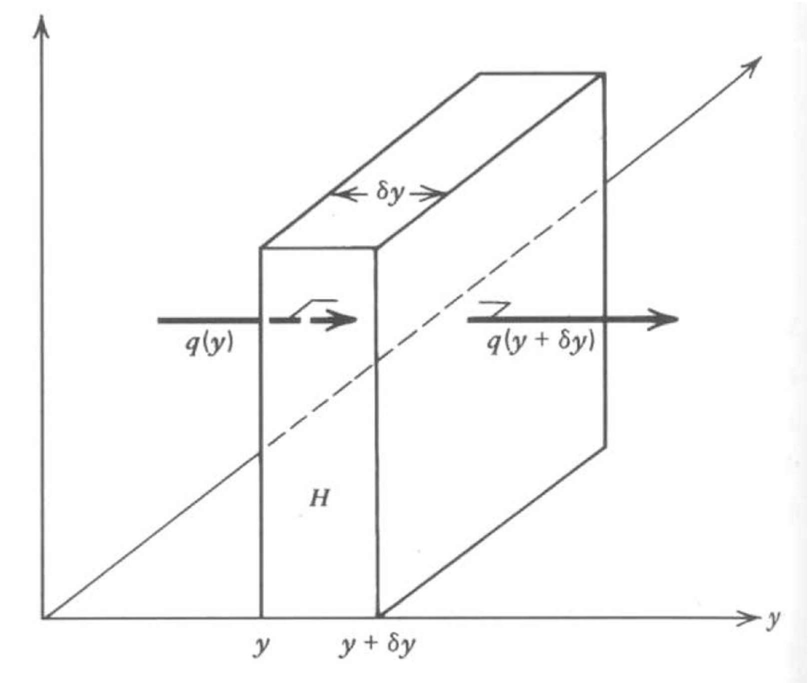
the plate

o the right

e per unit

$\Big] / \partial x \Big)$

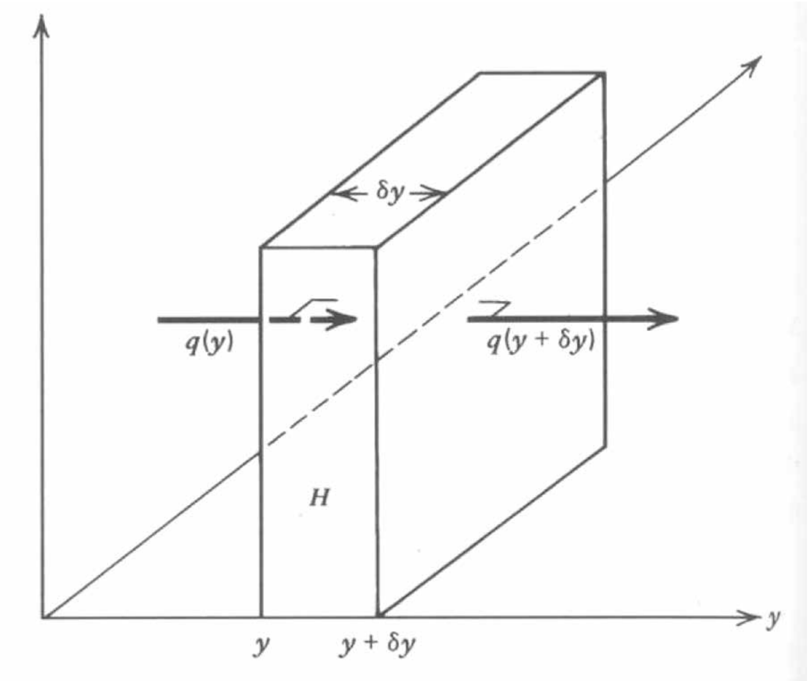
$\mathrm{d}x)$



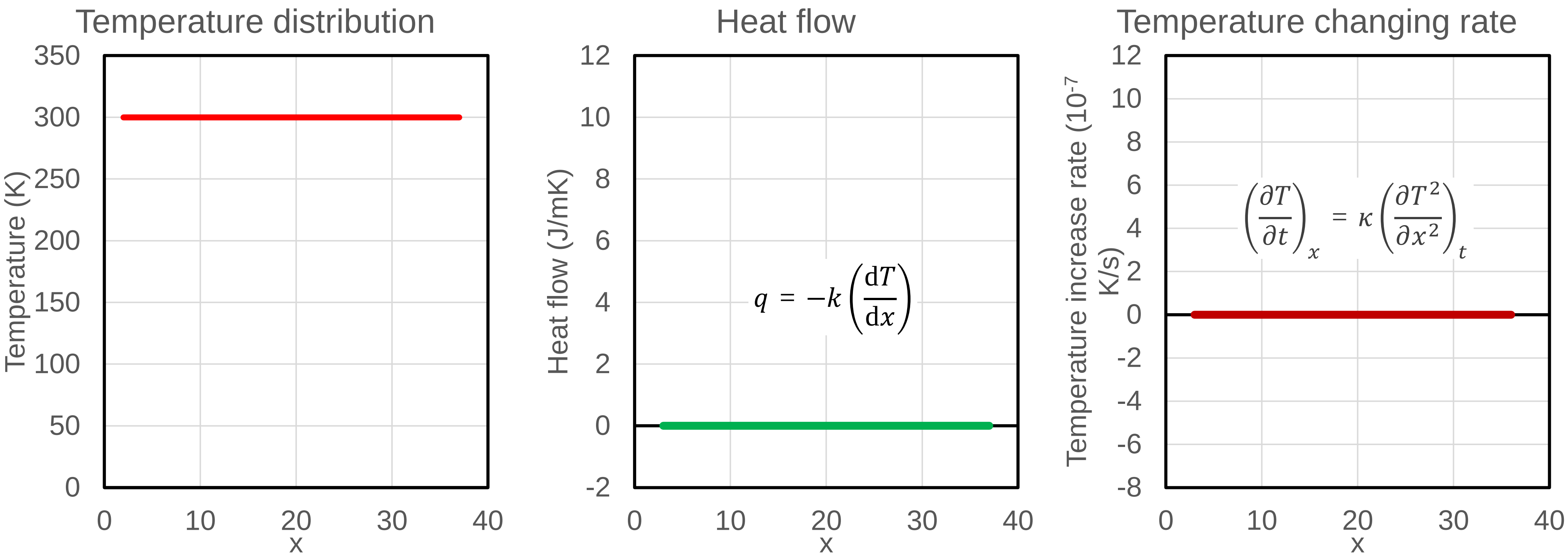
$$= k\delta_x S \left(\frac{\partial T^2}{\partial x^2}\right)_t \qquad (5.1.4)$$

Thermal diffusion -1

- T increasing rate at a position, $\left(\frac{\partial T}{\partial t}\right)_x$
  - $\left(\frac{\partial T}{\partial t}\right)_x = \frac{Q}{\rho C \delta_x S} = \frac{k \delta_x S}{\rho C \delta_x S}\left(\frac{\partial T^2}{\partial x^2}\right)_t$
    - (5.1.4) $Q = k \delta_x S \left(\frac{\partial T^2}{\partial x^2}\right)_t$
    - $\delta_x S$: volume of the plate
    - C: specific heat per weight
    - ρC: specific heat per volume
- Thermal diffusion equation
  - $\left(\frac{\partial T}{\partial t}\right)_x = \kappa \left(\frac{\partial T^2}{\partial x^2}\right)_t$ (5.1.5)
    - $\kappa = \frac{k}{\rho C}$: thermal diffusivity

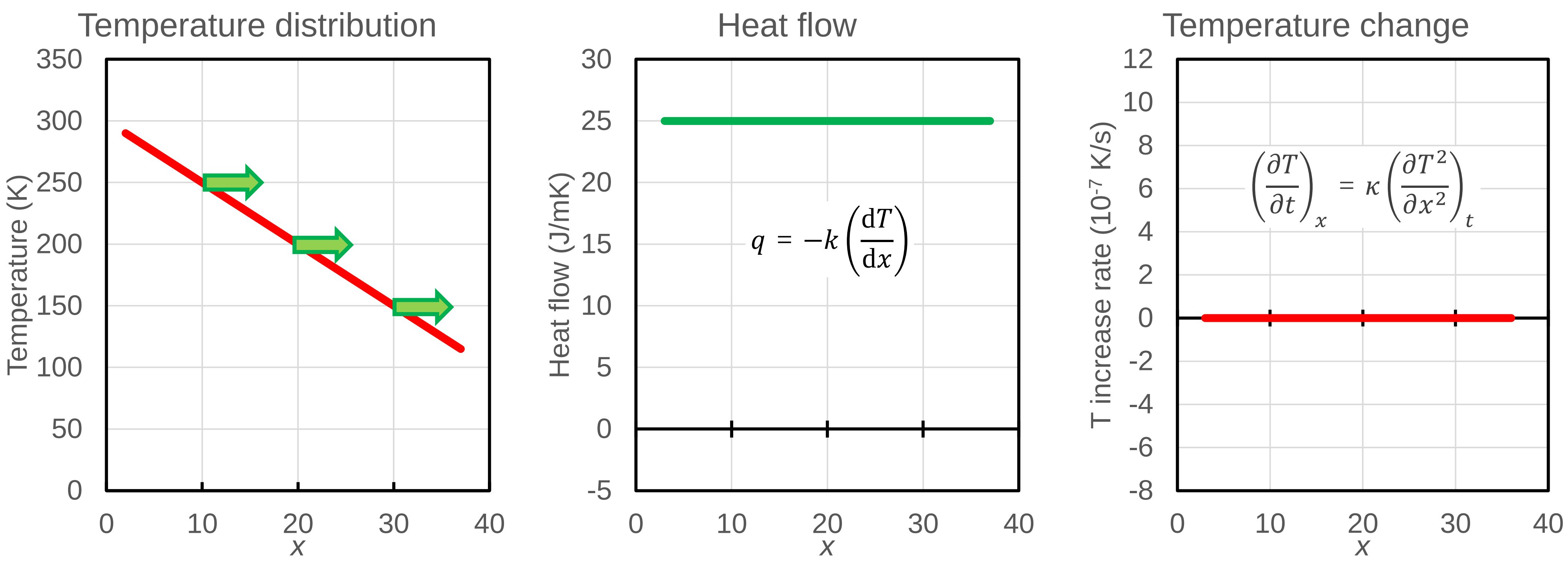


Heat flow and temperature change with time -1
Homogenous high temperature



- No heat flow
- No temperature change with time

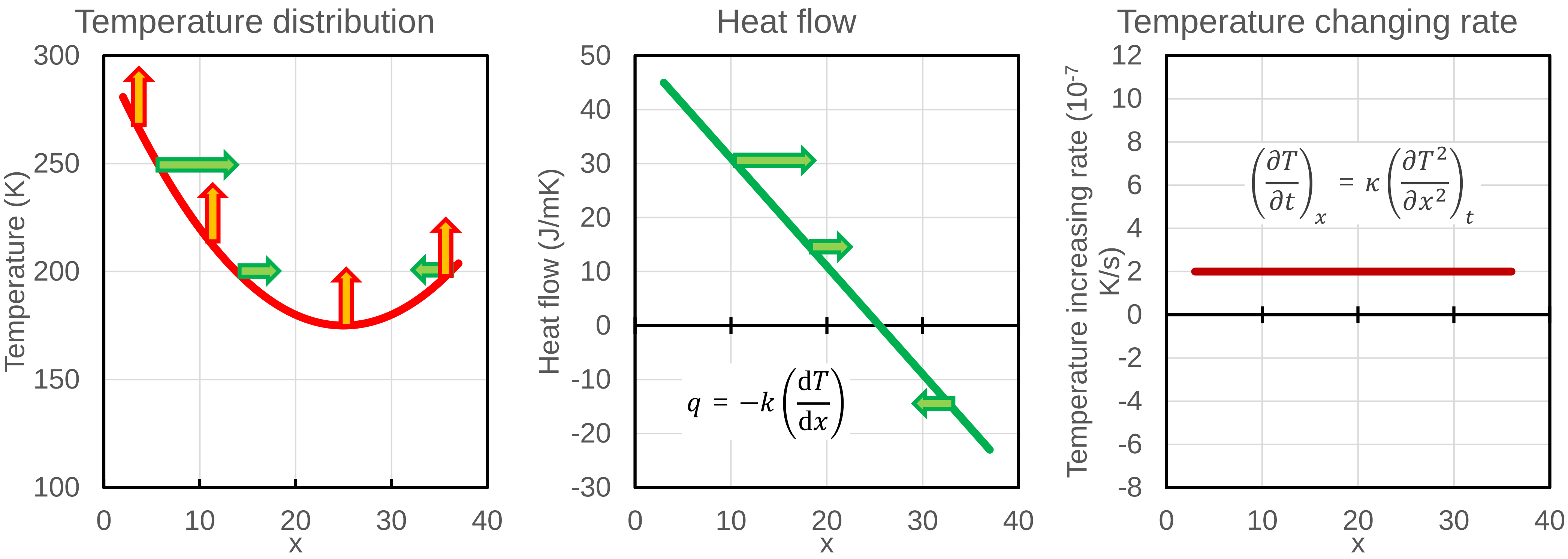
Heat flow and temperature change with time -2
Linear temperature gradient



q Homogeneous heat flow

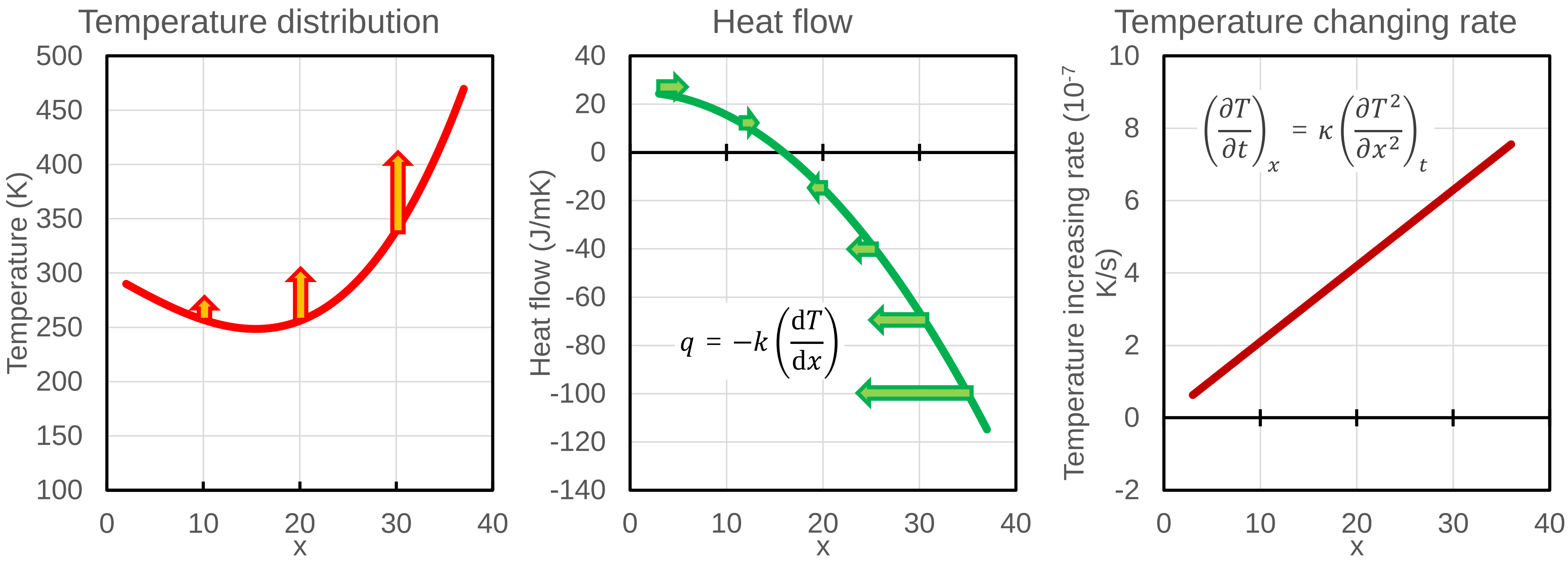
q No temperature change with time

Heat flow and temperature change with time -3
Parabolic temperature distribution



- Heat flow decreases to zero to the bottom of the temperature distribution.
- Temperature increases uniformly.

Heat flow and temperature change with time -4
Varying curvature with position



q The heat flow increase/decrease more significantly with increasing/decreasing curvature of the temperature distribution

q Temperature increases more rapidly with increasing the curvature of the temperature distribution.

Mineral Physics I

Chapter 5. Heat transfer properties

Section 1. Physics of heat transfer

End