Mineral Physics I
Chapter 4. Equation of State
Section 4. Murnaghan's EOS

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Derivation of Murnaghan's EOS -1

- Assumption:
  - Isothermal bulk modulus at high pressure K_T: linear function of P
    - ✓ $K_T = K_{T_0} + K_0' \cdot P$ (4.4.1)
- From Eq. (4.4.1) with the general definition of K_T (1.1.29) $K_T = -V\left(\frac{\partial P}{\partial V}\right)_T$
  - $K_{T_0} + K_0'P = -V\left(\frac{\partial P}{\partial V}\right)_T = -\frac{1}{\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T} = -\frac{1}{\left(\frac{\partial \ln V}{\partial P}\right)_T} = -\left(\frac{\partial P}{\partial \ln V}\right)_T$ (4.4.2)
  - $\left(\frac{\partial \ln V}{\partial P}\right)_T = -\frac{1}{K_{T_0}+K_0'P}$
  - $\ln V = \int\left(-\frac{1}{K_{T_0}+K_0'P}\right)\mathrm{d}P = -\int \frac{1}{\eta}\frac{\mathrm{d}P}{\mathrm{d}\eta}\mathrm{d}\eta,$
    - ✓ $\eta = K_{T_0} + K_0'P,\qquad \frac{\mathrm{d}P}{\mathrm{d}\eta} = \frac{1}{K_0'}$

    $= -\frac{1}{K_0'}\ln\left(K_{T_0} + K_0'P\right) + C$ $\qquad C$: integral constant (4.4.3)

Derivation of Murnaghan's EOS -2

- The integral constant C is obtained by $\ln V = -\frac{1}{K_0'}\ln\left(K_{T_0} + K_0'P\right) + C$ at $P = 0$
  - $\ln V_0 = -\frac{1}{K_0'}\ln K_{T_0} + C$
  - $C = \ln V_0 + \frac{\ln K_{T_0}}{K_0'}$ (4.4.4)
- Then, (4.4.3) $\ln V = -\frac{1}{K_0'}\ln\left(K_{T_0} + K_0'P\right) + C$ becomes:
  - $\ln V = -\frac{1}{K_0'}\ln\left(K_{T_0} + K_0'P\right) + \ln V_0 + \frac{\ln K_{T_0}}{K_0'}$
  - $K_0'\ln\left(\frac{V_0}{V}\right) + \ln K_{T_0} = \ln\left(K_{T_0} + K_0'P\right)$
  - $K_{T_0}\left(\frac{V_0}{V}\right)^{K_0'} = K_{T_0} + K_0'P$
  - $P = \frac{K_{T_0}}{K_0'}\left[\left(\frac{V_0}{V}\right)^{K_0'} - 1\right]$: Murnaghan's EOS (4.4.5)

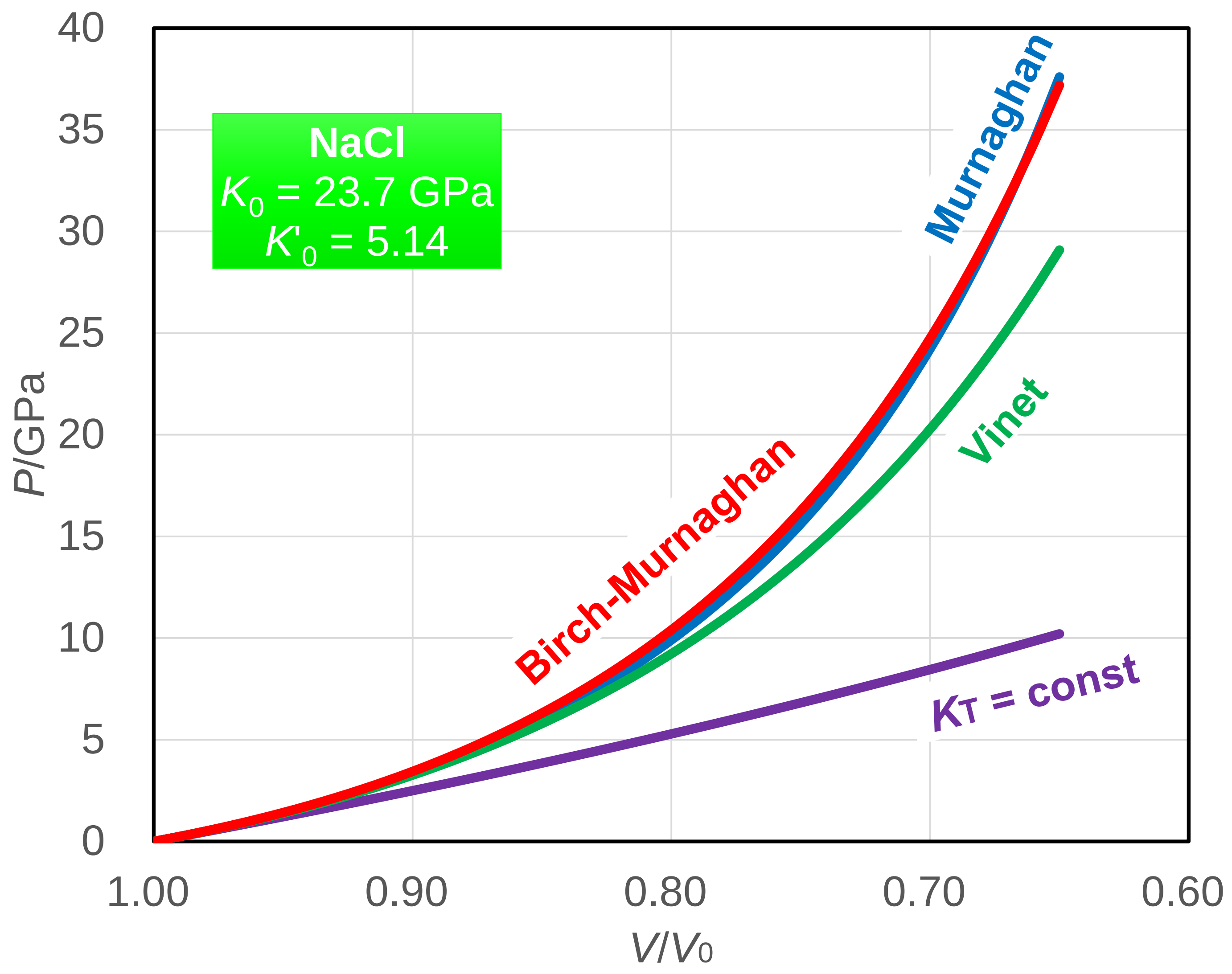
Compression of NaCl based on various EOS

Very low K_0 = 23.7 GPa

K'_0 larger than 4 (5.14)

negative K_0''

Relatively low pressure values even at high compression



Compression of MgO based on BM-EOS

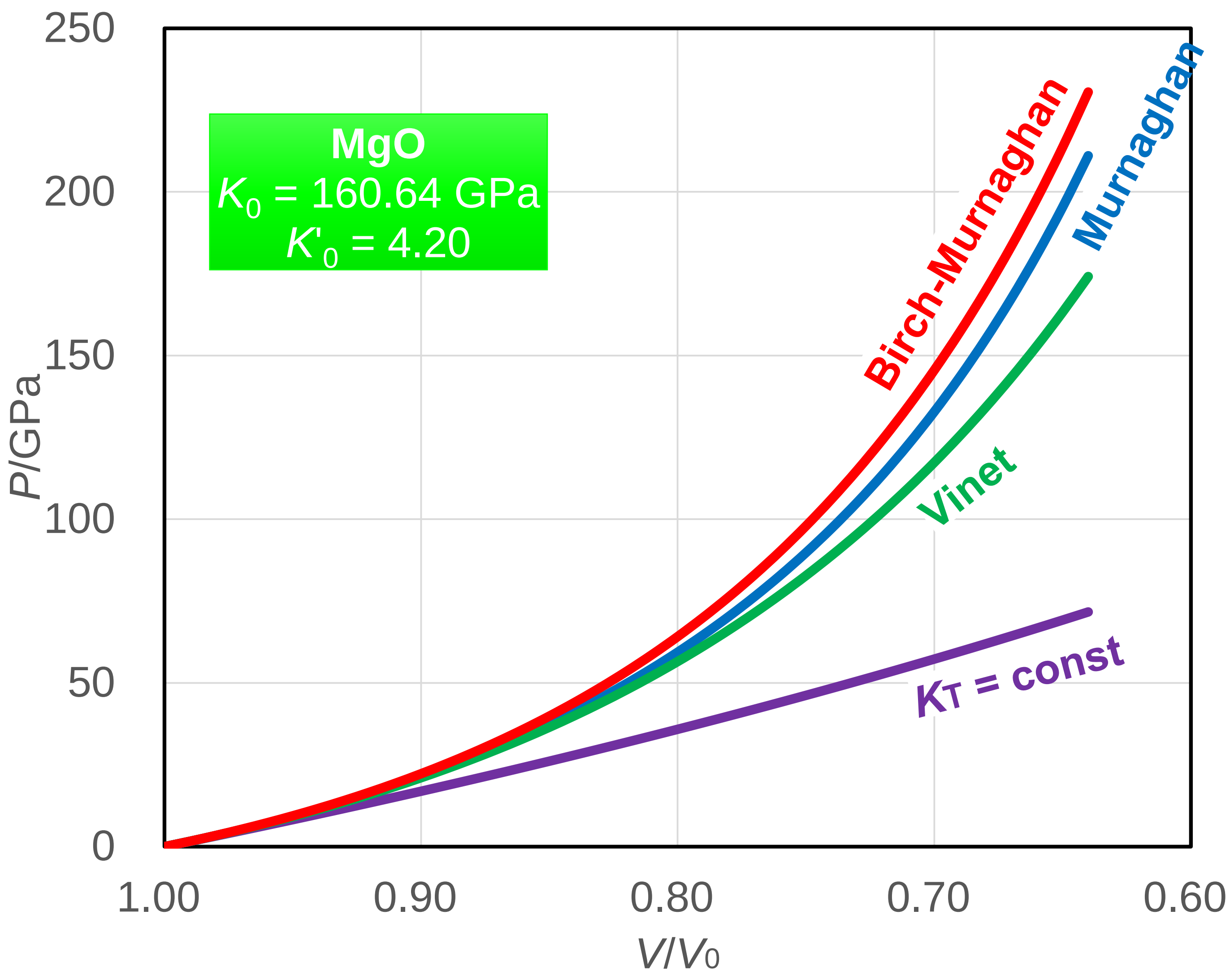
Relatively high K_0 = 160 GPa

K'_0 slightly larger than 4 (4.20)

No K_0" data

- K_0"=0 does not mean that 4^{th} EOS becomes 3^{rd} EOS

Relatively high pressure values by large compression



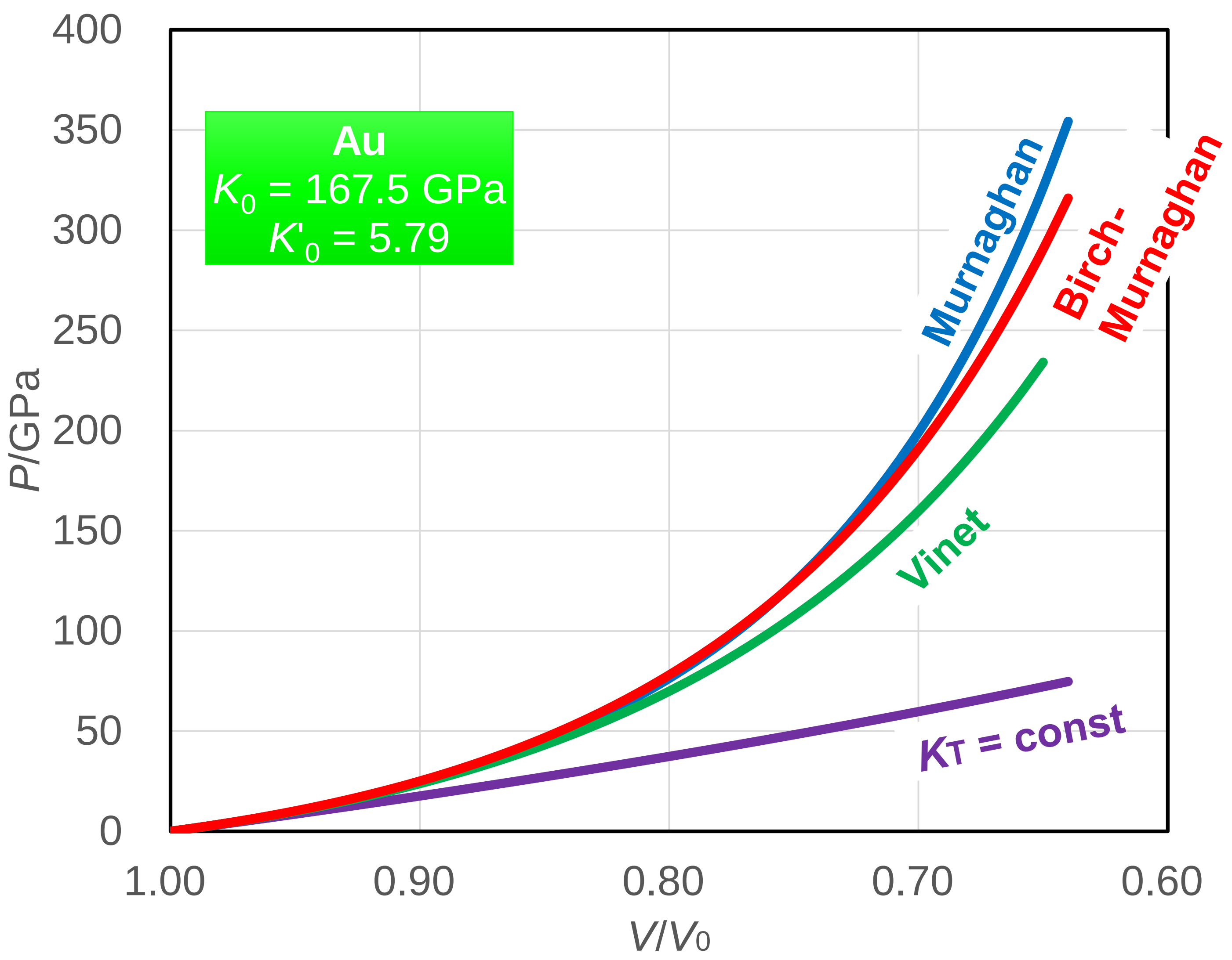
Compression of Au based on BM-EOS

Relatively high K_0 = 168 GPa

K'_0 larger than 4 (5.8)

No K_0“ data

Relatively high pressure values by large compression



Summary of comparison

- Birch-Murnaghan and Murnaghan's equations of states give similar pressures for the same compression
- Vinet equation of state gives lower pressures than Birch-Murnaghan and Murnaghan's equations of states
  - by 30% for NaCl, MgO and Au
- All of these equations of state give much higher pressures than the equations of state from the integration of the isothermal bulk modulus.

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